

# Chapter 9

## A Real Intertemporal Model with **Investment**

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### Already Done

- Consumer Behavior:
  - Work-Leisure choices (CHAPTER 4)
  - Intertemporal Consumption-Savings choices (CHAPTER 8)
- Production Side:
  - Firms' Production Technology and Labor Demand (CHAPTER 4)
  - Changes in Productivity affect  $c$ ,  $E$  and  $y$ . (CHAPTER 5)
- Government Side:
  - Government expenditure and the timing of taxes.

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### To do: REAL Model

- **REAL INTERTEMPORAL MODEL:**  
show how real aggregate output, real consumption, real investment, employment, real wage and the real interest rate are determined.
  
- CHAPTER 9: Investment behavior.

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## Investment Behavior

- **Determinants of Investment:**  
Study the microeconomic investment behavior of the firm, which makes an intertemporal decision regarding investment in the current period.
- Forgoes current profits to have higher capital stock and higher profits in the future.

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## Determinants of **high Investment**

- Lower capital stock.
- Higher expected future total factor productivity.
- Lower real interest rate.



KEY:  
opportunity cost  
of Investment

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## STUDY

- **Effects of:**
  - Government Spending Shock.
  - Total Factor Productivity Shock.
  - Capital Stock Shock.

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## MODEL

- Representative Consumer:
  - Supply labor and demand goods.
- Representative Firm:
  - Demand labor, supply goods and demand investment goods.
- Government:
  - Demand goods for purchases.

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## Consumer Budget

- CURRENT  $c + s = w(h-l) + \pi - T$
- FUTURE  $c' = w'(h-l') + \pi' - T' + (1+r)s$

- **LIFETIME**

$$c + c'/1+r = w(h-l) + \pi - T + (w'(h-l') + \pi' - T')/1+r$$

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## Consumer Problem

- Choose  $c, c', l$  and  $l'$
- Given  $w, w', r, T$  and  $T'$
- Cannot depict this on a single graph,
- Solution: describe consumer decision in terms of THREE marginal conditions (Chapter 4 and 8)

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## Three Marginal (Optimal) Decisions

- Work-leisure decision (CHAPTER 4):

$$MRS_{l,c} = w$$

Substitution between  $l$  and  $c$  is determined by  $w$

Remember: Income/Substitution effects of a change in  $w$

- Same in the future:

$$MRS_{l,c'} = w'$$

- Consumption-Savings decision (CHAPTER 8):

$$MRS_{c,c'} = 1 + r$$

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## NOTE

$w$

price of current leisure (labor) in terms of current  $c$

$w'$

price of future leisure in terms of future  $c'$

$1+r$

price of current consumption in terms of future consumption

$w(1+r)/w'$

current price of leisure relative to the future price of leisure

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## CONSUMER The Labor Supply

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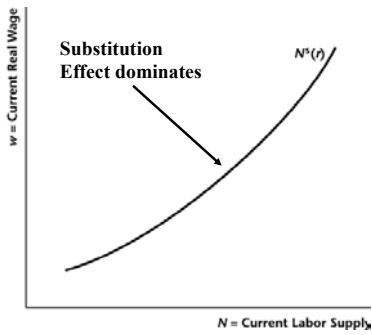
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Figure 9-1 The Representative Consumer's Current Labor Supply Curve

Why?  
We care for the Short-Run. In the Long-Run, both effects cancel out



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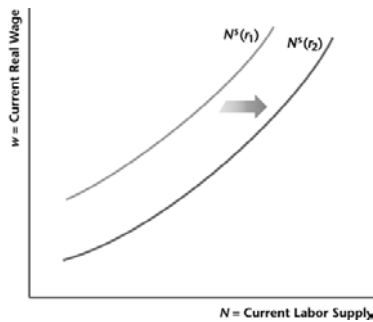
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Figure 9-2 An Increase in the Real Interest Rate Shifts the Current Labor Supply Curve to the Right

$\uparrow r \rightarrow \uparrow$  price of current leisure relative to future leisure

Intertemporal Substitution of leisure



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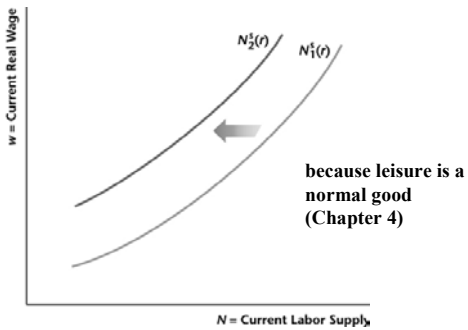
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Figure 9-3 Effects of an Increase in Lifetime Wealth



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## CONSUMER The Demand for Goods

Remember the FOUR games in chapter 8.

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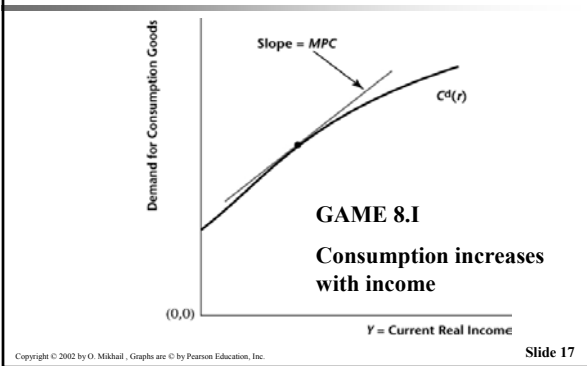
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Figure 9-4 The Representative Consumer's Current Demand for Consumption Goods Increases with Income



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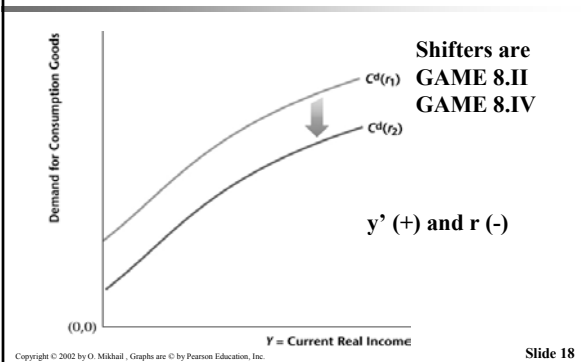
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Figure 9-5 An Increase in the Real Interest Rate from  $r_1$  to  $r_2$  Shifts the Demand for Consumption Goods Down



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**THE FIRM  
THE DEMAND FOR LABOR  
THE INVESTMENT DECISION**

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**THE FIRM  
The Labor DEMAND**

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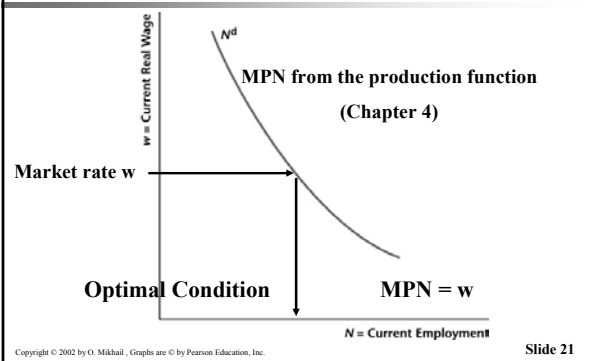
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Figure 9-7 The Demand Curve for Current Labor Is the Representative Firm's Marginal Product of Labor Schedule



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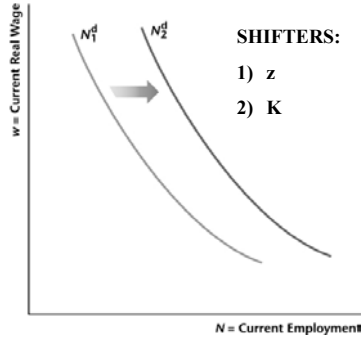
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Figure 9-8 The Current Demand Curve for Labor Shifts Due to Changes in Current Total Factor Productivity  $z$  and in the Current Capital Stock  $K$



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## THE FIRM The Investment Decision

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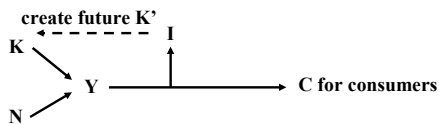
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## The Firm Production and Investment

- Production Function  $Y = z F(K,N)$
- Gross Investment  $I = K' - K + d K$
- Net Investment  $I^N = K' - K$
- Note that  $K' = (1 - d) K + I$



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## The Firm's Decision

- Maximize the Present Value of  $\pi$  by choosing  $N$ ,  $N'$  and  $I$ .
- i.e.,  $\max V = \max [ \pi + \pi' / 1+r ]$
- Note that
  - $\pi = Y - wN - I$
  - $\pi' = Y' - w'N' + (1 - d) K'$

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## The Investment Decision

- Marginal Cost = Marginal Benefit
- Marginal Cost = 1, an extra unit of Investment, reduces current profits by 1 unit, which reduces PV(profits) by 1.
- Marginal Benefit =  $(MPK' + 1 - d) / 1+r$  where  $MPK$  is the extra output from the extra unit of  $K$  and  $(1-d)$  is the left over  $K$ , all are discounted to the present.

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## Continue

Solve it  
 $MC = MB \rightarrow$   
OPTIMAL INVESTMENT RULE  
 $MPK' - d = r$   
Net Marginal Product  $K =$  real interest rate

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# GOVERNMENT

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## Government

Finances purchases through taxation and issuing bonds.

The Present Value Budget

$$G + G'/1+r = T + T'/1+r$$

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## Competitive Equilibrium

### BUILD THE MODEL

- The goods Market
- The labor Market

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**THE GOODS MARKET**

**CONSTRUCT**

**THE OUTPUT SUPPLY CURVE**

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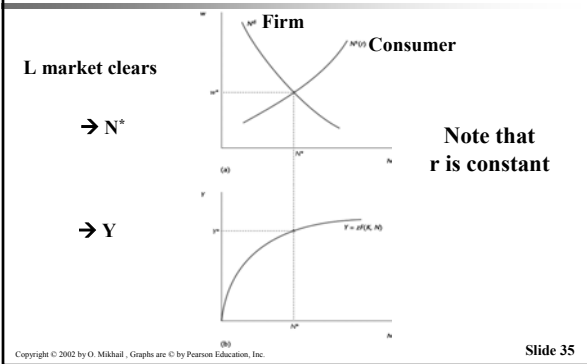
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Figure 9-11 Determination of Equilibrium in the Labor Market Given the Real Interest Rate  $r$




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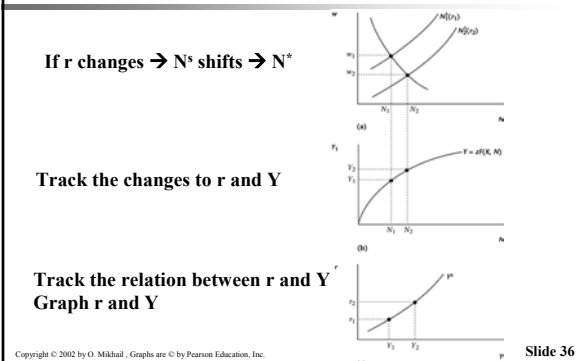
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Figure 9-12  
**Construction of the Output Supply Curve**




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## Shifts in the Output Supply

- Changes in Labor Supply  $\leftarrow$  Lifetime Wealth  $\leftarrow$  changes in  $G$  or  $G'$
- Changes in Labor Demand  $\leftarrow z$  or  $K$  (Slide 24)
- Changes in the Production Function  $\leftarrow z$  or  $K$

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## Figure 9-14 An Increase in Total Factor Productivity Shifts the $Y^S$ Curve

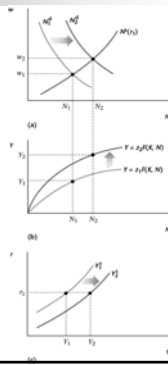
$\uparrow z$  or  $\uparrow K \rightarrow \uparrow MPN \rightarrow$

$\uparrow$  Labor Demand

$\uparrow N \rightarrow \uparrow Y$

[On NEW Production Function]

$\uparrow$  Supply of Output for the same  $r$



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## THE GOODS MARKET

CONSTRUCT  
THE OUTPUT DEMAND CURVE

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## Total current Aggregate Income Y

$$Y = C^d(r) + I^d(r) + G$$

- $C^d$  depends on  $r$ ,  $Y$  (moving on) and  $Y'$  (Chapter 8, Slide 17, 18)
- $I^d$  depends on  $r$  (moving on),  $z$  and  $K$  (Chapter 9, Slide 28 and 30)

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Slide 40

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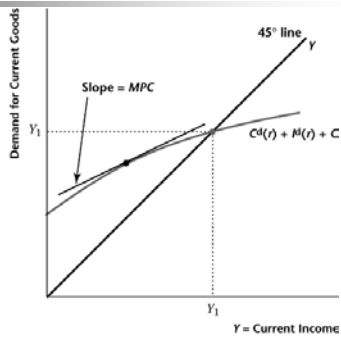
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## Figure 9-15 The Demand for Current Goods

Just added  
C to I and G



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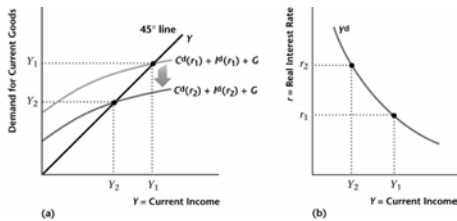
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## Figure 9-16 Construction of the Output Demand Curve



$\uparrow r \rightarrow \downarrow C$  (also  $\downarrow I$ )  $\rightarrow \downarrow Y$

Track  $r$  and  $Y$

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## Shifts of the Demand for Output

- Changes in Consumption (C)
- Changes in Investment (I)
- Changes in Government (G)
  
- Note that anything (other than  $r$ ) that will shift  $C^d$ ,  $I^d$  or  $G$  will also shift the  $Y^d$  (see **Slide 42**)

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## THE COMPLETE MODEL

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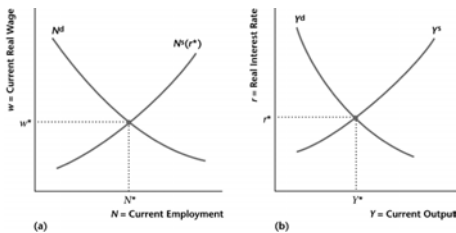
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Figure 9-18 The Complete Real Intertemporal Model



**SHIFTERS**

$z / K$   
slide 24

Wealth /  $r$   
slide 14

$C / I / G$   
slides 42 / 45

$N^s / N^d / z / K$   
slide 38

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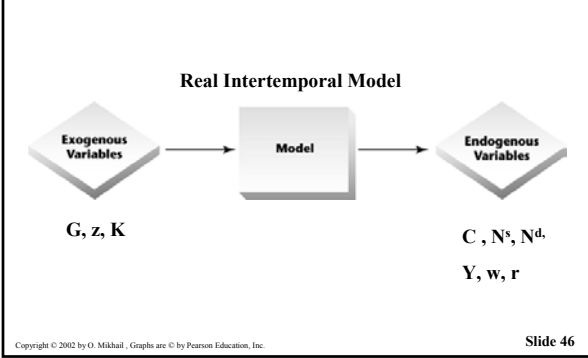
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# THE ONLY GAME




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## Key Messages of the Chapter

- Permanent vs. Temporary shock
  - Temporary: change in  $G$
  - Permanent: change in  $G$  and  $G^e$
  
- Expected in the future shock

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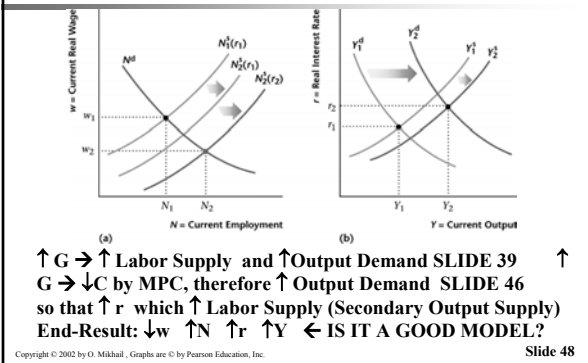
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**Figure 9-19 A Temporary Increase in Government Purchases**




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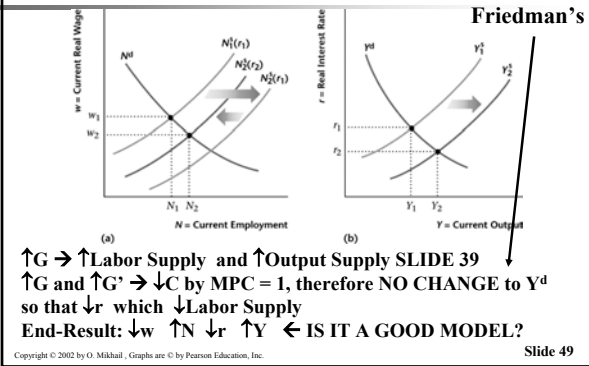
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Figure 9-20 A Permanent Increase in Government Purchases




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Permanent vs. Temporary Government shock

- Temporary: change in G  
 → **CROWDING-OUT** C and I
  
- Permanent: change in G and G'  
 → **NO CROWDING-OUT** I  
 → **stronger crowding-out** C  
 (because strong wealth effect)

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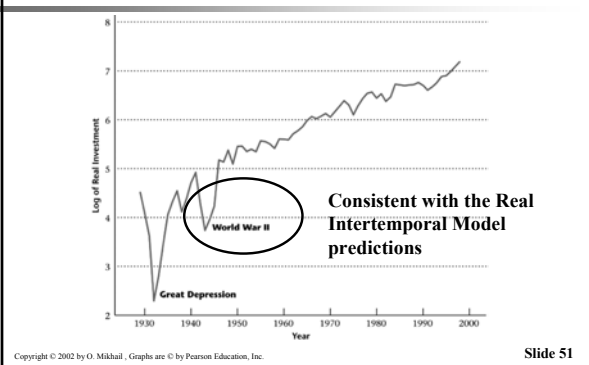
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Figure 9-21 Natural Log of Real Investment, 1929-1998




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## Real Interest Rate

### Model

→  $r$  increases when  $G$  increases which crowds-out Investment.

### Data

→ Investment decreased during WWII  
→ Did the  $r$  increase during WWII ?

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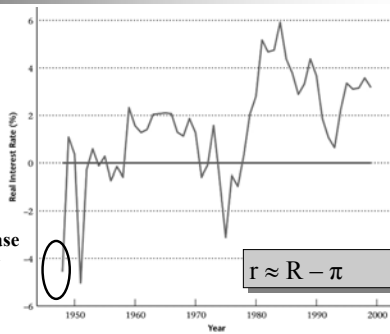
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## Real Interest Rate



**$r$  did not increase as predicted by the model**

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## Explanation

So why did Investment decrease when the real interest rate decreased during WWII ?

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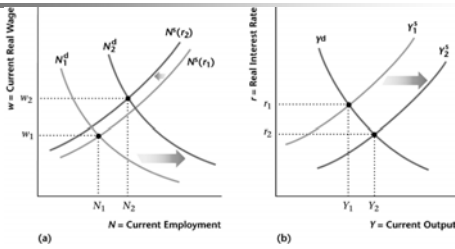
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Figure 9-23 The Equilibrium Effects of an Increase in Current Total Factor Productivity



$\uparrow z \Rightarrow \uparrow$  Labor Demand (MPN) and  $\uparrow$  Output Supply  
 so that  $\downarrow r$  which  $\downarrow$  Labor Supply (small)  
 End-Result:  $\uparrow w$   $\uparrow N$   $\downarrow r$   $\uparrow Y$   $\uparrow C$   $\uparrow I$

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Real Business Cycle Theory

Temporary shocks to  $z$  are a candidate as a cause of business cycles

Increases in the relative price of energy  
 (decreases in total factor productivity)

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