

Chapter 4

Future Value, Present Value and Interest Rates

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Future Value

Future Value is the value on some future date of an investment made today.

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Future Value

$$\text{\$100} + \text{\$100}(0.05) = \text{\$105}$$

$$\text{PV} + \text{Interest} = \text{FV}$$

$$\text{PV} + \text{PV} \cdot i = \text{FV}$$

PV = Present Value

FV = Future Value

i = interest rate (as a percentage)

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Future Value

Future Value in one year.

$$FV = PV*(1+i)$$

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Future Value (compound i)

Future Value in two years

$$\$100 + \$100(0.05) + \$100(0.05) + \$5(0.05) = \$110.25$$

Present Value of the Initial Investment
+ Interest on the initial investment in the 1st Year
+ Interest on the initial investment in the 2nd Year
+ Interest on the Interest from the 1st Year in the 2nd Year
= Future Value in Two Years

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Future Value

General Formula – Future value of an investment of PV in n years at interest rate i (measured as a decimal, or 5% = .05)

$$FV_n = PV*(1+i)^n$$

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Future Value

Table 4.1 Computing the Future Value of \$100 at 5% Annual Interest

Years into Future	Computation	Future Value
1	$\$100(1.05)$	\$105.00
2	$\$100(1.05)^2$	\$110.25
3	$\$100(1.05)^3$	\$115.76
4	$\$100(1.05)^4$	\$121.55
5	$\$100(1.05)^5$	\$127.63
10	$\$100(1.05)^{10}$	\$162.89

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Future Value

Note:

Both n and i must be measured in same time units—if i is annual, then n must be in years, So future value of \$100 in 18 months at 5% is

$$FV = 100 \cdot (1 + .05)^{1.5}$$

Month: $n = 1/12$
 Day: $n = 1/365$
 A basis point = 0.01

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Present Value

Present Value (PV) is the value today (in the present) of a payment that is promised to be made in the future.

OR

Present Value is the amount that must be invested today in order to realize a specific amount on a given future date.

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Present Value

Present Value of an amount received in one year.

Solving the Future Value Equation

$$FV = PV*(1+i)$$
$$PV = \frac{FV}{(1+i)}$$

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Present Value

Example:

\$100 received in one year, $i=5\%$

$$PV = \$100 / (1 + .05) = \$95.24$$

Note:

$$FV = PV*(1+i) = \$95.24*(1.05) = \$100$$

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Present Value

Present Value of \$100 received n years in the future:

$$PV = \frac{FV}{(1+i)^n}$$

The most important relationship in studying financial instruments

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Present Value

Example

Present Value of \$100 received in 2 ½ years and an interest rate of 8%.

$$PV = \$100 / (1.08)^{2.5} = \$82.50$$

Note:

$$FV = \$82.50 * (1.08)^{2.5} = \$100$$

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Present Value

Important Properties of Present Value

Present Value is higher:

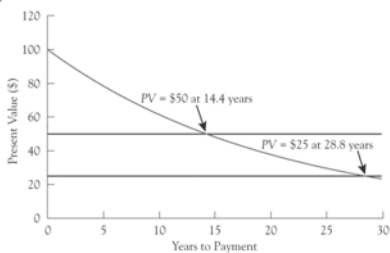
1. The higher the future value (FV) of the payment.
2. The shorter the time period until payment. (n) [Figure 4.1]
3. The lower the interest rate (i) [Figure 4.2]

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Present Value

Figure 4.1 Present Value of \$100 at 5% Interest

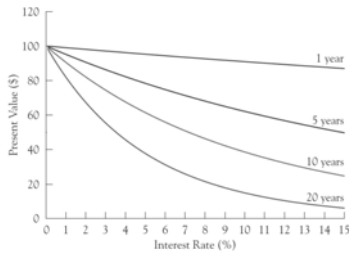


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Present Value

Figure 4.2 The Relationship between Present Value and Interest Rates



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Internal Rate of Return

The Internal Rate of Return is the interest rate that equates the present value of an investment with its cost.

Why useful?

- To compare across investment projects
- To invest or not to invest

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Internal Rate of Return

A machine with a price of \$1,000,000 that generates \$150,000/year for 10 years.

$$\$1,000,000 = \frac{\$150,000}{(1+i)^1} + \frac{\$150,000}{(1+i)^2} + \frac{\$150,000}{(1+i)^3} + \dots + \frac{\$150,000}{(1+i)^{10}}$$

Solving for i , $i = .0814$ or 8.14%

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Bond Pricing

A bond is a promise to make a series of payments on specific future date.

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Bond Pricing

The price of a bond is the Present Value of its payments.

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Bond Pricing

Payment stops at the maturity date. (n)

A payment is for the face value (F) or principle of the bond

Coupon Bonds make annual payments called, Coupon Payments (C), based upon an interest rate, the coupon rate (i_c),

$$C = i_c * F$$

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Bond Pricing

A bond that has a \$100 principle payment in n years. The present value (P_{BP}) of this is now:

$$P_{BP} = \frac{F}{(1+i)^n} = \frac{\$100}{(1+i)^n}$$

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Bond Pricing

If the bond has n coupon payments (C), where $C = i_c * F$, the Present Value (P_{CP}) of the coupon payments is:

$$P_{CP} = \frac{C}{(1+i)^1} + \frac{C}{(1+i)^2} + \frac{C}{(1+i)^3} + \dots + \frac{C}{(1+i)^n}$$

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Bond Pricing

Present Value of Coupon Bond (P_{CB}) =
Present value of Yearly Coupon Payments
(P_{CP}) +
Present Value of the Principal Payment
(P_{BP})

$$P_{CB} = P_{CP} + P_{BP} = \left[\frac{C}{(1+i)^1} + \frac{C}{(1+i)^2} + \frac{C}{(1+i)^3} + \dots + \frac{C}{(1+i)^n} \right] + \frac{F}{(1+i)^n}$$

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Bond Pricing

Note:

The price of a bond and the interest rate are inversely related -- higher interest rate gives lower bond prices.

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Real and Nominal Interest Rates

Nominal Interest Rates (i)

Interest Rates expressed in current dollar terms.

Real Interest Rates (r)

Nominal Interest Rate adjusted for inflation.

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Real and Nominal Interest Rates

Fisher Equation:

$$i = r + \pi^e$$

or

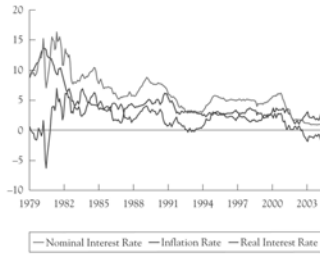
$$r = i - \pi^e$$

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Real and Nominal Interest Rates

Figure 4.3 Nominal Interest Rates, Inflation, and Real Interest Rates





Real and Nominal Interest Rates

Figure 4.4 Inflation and Nominal Interest Rates, April 2004



Chapter 4

End of Chapter
