Real-Business-Cycle Theory: Elasticites, De-trending, Criticisms and Failures Class Notes for ECO 6206

O. Mikhail

October 2, 2003

1 Elasticities

Since the extent of the effect of a shock on employment depends on the intertemporal substitution of labour supply, this subsection defines and emphasizes the role of elasticities in the baseline RBC model. For the general momentary utility function,

$$U(C,L) = V(C) \bullet C(L) = \frac{1}{1-\sigma}C^{1-\sigma} \bullet \frac{1}{1-\sigma_1}L^{1-\sigma_1} \tag{1}$$

We have,

$$\begin{array}{ll} U_C = C^{-\sigma}V(L) > 0 & U_L = L^{-\sigma_1}V(C) > 0 \\ U_{CC} = -\sigma C^{-1-\sigma}V(L) < 0 & U_{LL} = -\sigma_1 L^{-1-\sigma_1}V(C) < 0 \\ U_{CL} = U_{LC} = C^{-\sigma}L^{-\sigma_1} > 0 & \end{array} \tag{2}$$

The elasticities of marginal utility U_C with respect to C and L are,

$$\zeta_{U_cC} = \frac{U_{CC}C}{U_C} = \frac{-\sigma C^{-1-\sigma}V(L)C}{C^{-\sigma}V(L)} = -\sigma \tag{3}$$

$$\zeta_{U_cL} = \frac{U_{CL}L}{U_C} = \frac{C^{-\sigma}L^{-\sigma_1}L}{C^{-\sigma}\frac{1}{1-\sigma_1}L^{1-\sigma_1}} = 1-\sigma_1 \tag{4}$$

The intertemporal elasticity of substitution in consumption equals σ. As σ increases (approaches 1, i.e. logarithmic), the decrease in U_C is more rapid in response to an increase in C, and the consumer is less willing to accept deviations from a uniform pattern of consumption.

The elasticities of marginal utility U_L with respect to C and L are,

$$\zeta_{U_LC} = \frac{U_{LC}C}{U_L} = \frac{C^{-\sigma}L^{-\sigma_1}C}{L^{-\sigma_1}\frac{1}{1-\sigma}C^{1-\sigma}} = 1-\sigma \tag{5}$$

The intertemporal elasticity of substitution in leisure equals σ_1, as shown by,

$$\zeta_{U_LL} = \frac{U_{LL}L}{U_L} = \frac{-\sigma_1 L^{-1-\sigma_1}V(C)L}{L^{-\sigma_1}V(C)} = -\sigma_1 \tag{6}$$

1.1 The Frisch Elasticity of Labour Supply

It is useful to consider the λ-constant or Frisch labour supply. Let λ be the Lagrangian multiplier associated with the worker's intertemporal budget constraint. The first-order condition associated with the labour supply is,

$$\frac{\partial U(n_1, ..., n_t, ...n_T)}{\partial n_t} = \lambda w_t \tag{7}$$

where w_t denotes the real wage in period t stated in period 0 prices (discounted to period 0). The Frisch inverse labour supply function is the marginal disutility of work stated in wage units:

$$\frac{1}{\lambda} \cdot \frac{\partial U(n_1, ..., n_t, ...n_T)}{\partial n_t} \tag{8}$$

When U is additively separable in labour, this can be solved to simplify for the labour supply as a function of the current wage. When U is not additively separable, the supply price of work in one period is a function of the level of work in that and other periods.

The elasticity of the labour supply schedule is

$$\zeta = \sigma_1 \cdot \frac{\overline{n} - n}{n} \tag{9}$$

It is equal to the intertemporal elasticity of substitution in leisure, σ_1, multiplied by the ratio of non-work time to work time. The elasticity ζ controls labour supply over the life cycle. If the wage rate were to double (fully anticipated by the worker at age 20) over the same period, a worker with an ζ of 1 will work twice as many weeks at age 40 as at age 20. Empirical evidence points to ζ being near the values 0.1 to 0.2. A larger Frisch elasticity generates larger responses to economic shocks in equilibrium models, since agents are more willing to substitute leisure across time.

For the utility kernel $(1-\phi)\log C_t + \phi\log(1-n_t)$, the Frisch elasticity of labour supply equals $(1-n)/n$, the steady-state ratio of leisure to labour, or $\phi/(1-\phi)$. The intertemporal elasticity of leisure is equal to 1. A 1% change in leisure results in $\frac{\phi}{1-\phi}$% change in hours of employment. This kernel is often criticized that its labour supply elasticity is much higher than that of prime age males estimated from panel data.

Christiano and Eichenbaum (1992) used a range of 3 to 5 for the Frisch elasticity. They estimated ϕ to be equal to 5/6. Prescott (1986) choose a value for ϕ closer to 2/3, but typically magnifies this elasticity by allowing past values of leisure to enter into the utility function. A value of 2/3 means that 2/3 of the time is allocated to non-market activities. Swanson (1999b) used a value of 1.7/3.

Lloyd and Niemi (1979) investigated if the labour supply elasticity shifted over time, and for which demographic groups it did. Using quarterly U.S.A. data,[1] the study found evidence of statistically significant shifts - from the period

[1] Source : *Employment and Earnings.*

1956-1965 to 1966-1976 - in the labour supply elasticity. The most significant shift was due to sectoral shifts in demand, unfavourable to men and favourable to women (i.e., increased female participation rates).

2 De-trending

Lucas' (1977) definition of the term 'business cycle' requires detrending the business cycle data. If 'business cycle' fluctuations are defined as deviations around a trend, then a natural first step to examine the fluctuations is to de-trend the data. One way of eliminating the trend is to use the Hodrick-Prescott filter (Hodrick and Prescott 1980).

The first step of the HP curve-fitting method is to take the logarithms of the variables for two reasons: 1) to compress the units in which the variables are measured in, and 2) because of the inherent exponential trend in most aggregate economic variables. The selected trend path $\{\tau_t\}$ is one which minimizes the sum of squared deviations from a given series $\{Y_t\}$ subject to the constraint that the sum of the squared sum differences not be large. Formally,

$$\min_{\{\tau_t\}_{t=1}^{T}} \sum_{t=1}^{T} (Y_t - \tau_t)^2 \tag{10}$$

subject to

$$\sum_{t=2}^{T-1} \left[(\tau_{t+1} - \tau_t) - (\tau_t - \tau_{t-1})\right]^2 \leq \mu \tag{11}$$

where μ is a parameter governing the smoothness of the trend. The smaller μ is, the smoother it is. If $\mu = 0$, the least squares time trend is linear. Usually, μ is set so that the Lagrangian multiplier λ of the constraint equals 1600. When the observation period is in quarterly frequency, this produces the appropriate degree of smoothness. Therefore, the minimization problem reduces to

$$\min_{\{\tau_t\}_{t=1}^{T}} \sum_{t=1}^{T} (Y_t - \tau_t)^2 + \lambda \cdot \sum_{t=2}^{T-1} \left[(\tau_{t+1} - \tau_t) - (\tau_t - \tau_{t-1})\right]^2 \tag{12}$$

$$\min_{\{\tau_t\}_{t=1}^{T}} \sum_{t=1}^{T} (Y_t - \tau_t)^2 + 1600 \cdot \sum_{t=2}^{T-1} \left[(\tau_{t+1} - \tau_t) - (\tau_t - \tau_{t-1})\right]^2 \tag{13}$$

The second sum of the squared term is an approximation of the derivative of τ_t at time t. One attempts to minimize two sums of squares: the sum of squared cyclical residuals and the sum of squared $\Delta^2\tau_t$. The smoothing parameter λ gives relative weight to these two sums of squares.[2]This parameter acts as a

[2]The rationale for setting $\lambda = 1600$ is as follows. The parameter $\lambda = \sigma_c^2/\sigma_\tau^2$, where σ_c^2 denotes the variance of the cyclical component and σ_τ^2 denotes the variance of the trend component. Hodrick and Prescott used "... the prior view that a five percent cyclical component is moderately as large as is one-eight of one percent change in the rate of growth in a quarter ...". Therefore, $\lambda^{1/2} = \frac{5/1}{1/8}$ or $\lambda = 1600$ as a value for the smoothing parameter.

penalty for the acceleration of growth. Finally, the deviations from trend are computed as,

$$Y_t^d = Y_t - \tau_t \qquad \text{for} \quad t = 1, ...T \tag{14}$$

The HP filter is a high band pass filter that eliminates all frequencies of 32 quarters (8 years) or greater. It decomposes the macroeconomic time series into a nonstationary trend component and a stationary cyclical component. Over the past nineteen years, the HP filter became the standard practice to detrend and the hallmark of real business cycle models.

Proponents of the use of the HP filter often explain that it is just a computational procedure used to fit a smooth curve through the data, i.e., that 'it is just a curve-fitting technique'. Opponents of the use of the HP filter have shown that the filter distorts the dynamic properties of the data. The filter is responsible for generating spurious business cycle periodicity when there is no cycle present in the original data (see Cogley and Nason (1995)). Also, King and Rebelo (1993) provided examples in which the use of HP filter alters substantially measures of persistence, variability and co-movements of economic time series data. They advocated the implementation of a trend component in RBC models to eliminate the use of any filtering.

There are also other detrending methods in the literature. For example, Lucas (1980b) employed an exponential smoothing filter (ES) in his investigation of the quantity theory of money. The ES filter solves a minimization problem similar to the HP filter. It is

$$\min_{\{\tau_t\}_{t=1}^T} \sum_{t=1}^{T} (Y_t - \tau_t)^2 + \lambda \cdot \sum_{t=2}^{T-1} \left[(\tau_t - \tau_{t-1})\right]^2 \tag{15}$$

Note that the parameter λ here penalizes for the changes in the growth component.

3 Criticisms

For a complete review of RBC controversies, see the series of discussion papers in The Economic Journal (1995). In my view, criticisms of RBC models are classified as ideological, methodological, end-result and goodness-of-fit.

Ideologically, critics attack the built-in Walrasian market clearing foundation as a way of describing markets behaviour, especially that of the labour market. Many economists object to the notion of agents' intertemporal decisions to generate a labour supply. Their argument is as follows. Explaining the Great Depression on the basis that the labour supply is the product of agents' intertemporal decisions, is likely to be unrealistic. To explain the Great Depression using such decisions, the assumption has to be that agents anticipated WWII a decade prior to its start and decided to hold off their supply for labour until the increase for demand generated by WWII. Such a voluntary-unemployment explanation during 1930s is unreasonable. In his criticism, Stiglitz (1986) questioned also capital (i.e., machines) unemployment during the same era.

Methodologically, the criticisms were about the objectivity versus the subjectivity of the calibrating exercise. The use of Solow residuals as impetus came under heavy criticism. The criticisms of the end-result of RBC point to the models' inability to reproduce certain stylised facts such as: variability of employment exceeding that of productivity, the instantaneous correlation between employment and productivity close to zero and average productivity that leads the cycle.[3]Goodness-of-fit criticisms highlighted and strongly condemned the ad-hoc method(s) of judging the merits of each model. The absence of a metric, by which one measures how good is the model as an approximation to the business cycle data, is still a topic of research. Also, the absence of formal statistical tests led many to label the RBC as 'unworthy' of acceptance.

The success of RBC modeling in explaining business cycles is still a question open to debate. However, Eichenbaum (1995, p.1609) reiterated - in defense of the brittleness of RBC - that "We do not need high power econometrics to tell us that models are false. We know that. What we need are interesting diagnostic tools to help us understand the dimensions along which misspecified models do well and the dimensions along which they do poorly".

The real business cycle literature shows that if one is to reconcile the cyclical and persistent pattern of the data with a general equilibrium stochastic macroeconomic model, one must use the same pattern in the 'productivity shocks' that drive the model impulse responses.[4]Empirical measures of aggregate technology are obtained by calculating the Solow (1957) residuals. However, the standard deviation for the U.S.A. Solow residual equals 0.763, while the standard deviation of Gross National Product (GNP) is 1.8 percent. How can one use the 'productivity shocks' (measured by the Solow residual) pattern to drive the model impulse responses and get a result of 1.8 percent variability for GNP?

Cogley and Nason (1993) elaborate on this point. They showed that in a typical (baseline) RBC model,[5]output dynamics are determined by impulse dynamics. In other words, the output series generated from the artificial model is represented as a filtered transformation of the external shocks to the model. For example, if the shock is an AR(1) process, then output is an ARMA(3,2) process. In brief, external shocks completely drive the model's generated output series, pointing out how weak is the propagation mechanism of typical RBC models. The output dynamic properties are only a reflection of the impulse dynamic properties. There are just not enough dynamics (the propagation mechanism is very weak) in the typical RBC models since the output dynamic properties are completely dominated by impulse dynamics.

3.1 Goodness-of-fit

Real Business Cycle research has often relied on matching unconditional second moments from the data in the real economy with unconditional second moments

[3]Usually referred to as 'labour productivity cycle' in the literature.

[4]This is usually referred to as 'The baseline real business cycle model' in the literature.

[5]The Cogley and Nason (1993) typical RBC model is in the appendix, as well as the parameters values used in their study.

from the data generated by an artificial model economy. Such an approach to assess the model's goodness-of-fit was heavily criticized and labeled - by many - as 'the eye-ball metric'. A classical alternative was suggested by Watson (1993). This study developed a goodness-of-fit measure for the class of dynamic econometric models in which all the endogenous variables are covariance stationary. In this context, the economic model is an abstraction of the real economy and is viewed as an approximation to the stochastic process generating the data. To measure the quality of this approximation, Watson proposed a measure of goodness-of-fit motivated by models of measurement errors in the Slutsky (1927) spirit. His approach was to quantify how much stochastic error must be added to the model's variables so that the model's artificial second moments do match the real economy' moments. This treats the discrepancy between the model and data as a stochastic process. Once this error is computed, one can construct a measure of fit from its size. This approach to minimizing the approximation error, in a sense, mirrors the R^2 in simple linear regression.

The criticisms of Watson' procedure are: 1) it can not account for moments other than the second ones and 2) nonlinearities and variations in conditional second moments (such as ARCH type time series) are ignored for simplicity. Another criticism is on how the procedure views the parameters. In the usual calibration exercises parameter values are viewed as a point-mass priors around the values.

A different procedure was proposed by Bayesian analysis. DeJong et al (1996) proposed a Bayesian approach to deal with the parametric uncertainty. By specifying a prior distribution over the parameter values, one can generate a distribution over the statistical properties of the simulated artificial data. In the case of the typical RBC model, this procedure concluded that modest prior specification is the road to take.

In general, the ratio of the standard deviations of aggregate hours to those of output has been emphasized in the literature as a measure of the simulated model economy's goodness of fit (see Kydland and Prescott 1982 and Hansen 1985). It is 1.47 for the U.S.A. data.

4 RBC failures

This section focuses on RBC failures to account for observed employment variability and output persistence.

4.1 Observed employment volatility

Prescott (1986) reported that observed employment is twice as volatile as the one simulated from the standard RBC economy. In the U.S.A. data, the variance of hours worked relative to the variance of output equals 0.95 percent. A usual RBC baseline model generates a ratio of 0.52 percent. Most RBC models generate a substantially smaller volatility in employment than that in the data.

Campbell (1994) investigated this issue. The study found that a one percent shock, decreasing technology, lowered employment by 0.45 percent in the baseline RBC model. Therefore, to explain a decline of three percent employment in recession, one must assume a seven percent decrease in technology, a number which is obviously unrealistic. For Europe, employment did not rise during the 1970-1985 period, although total factor productivity increased more than twice as much as it did in the U.S.A. Failure of RBC models to generate matching employment variability sparked wide interest among researchers and led to a search for alternatives that could explain this observation. Examples included: indivisible labour, nominal wage contracts and labour market search.

Fraisie and Langot (1994, p. 1581) asked the same question: "Can RBC models be saved?" They considered a model with indivisible labour, labour hoarding and adjustment costs. They concluded that "... the introduction of labor adjustment costs is a necessary condition for the model to reproduce a productivity cycle ..." (p. 1582). However, they also concluded that labour hoarding is a necessary assumption to achieve a one period gap between productivity and employment (in business cycle data, employment is coincident and productivity is leading).

4.2 Persistence in aggregate output

Cogley and Nason (1995) concluded that actual output dynamics are more persistent than those generated from standard RBC models. Since the baseline KPR model is driven only by the single technology shock, the persistence of the output, consumption and investment depended heavily on the persistence assumption used in the technology shock.

The baseline model fails to account for the heterogeneity of the workers or jobs. It does not contain incentives for a worker to change jobs and no suggestion that a worker might be more productive in the new job than the current one. The focus here is on the movement of workers from unsuccessful productive units to growing ones.