O. Mikhail – ~~Class Notes~~ Soln to Assign II – ECO 6206
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Soln to Assign II

Question 1

$$\dot{k} = s f(k) - (n+g+\delta)k$$

let $f(k) = k^{\alpha}$ (Cobb-Douglas)

$$\longrightarrow \dot{k} = s k^{\alpha} - (n+g+\delta)k$$

at st st k^* $\longrightarrow$ $\dot{k} = 0$ $\longrightarrow$ $s k^{*\alpha} = (n+g+\delta)k^*$

solve for k^*

$$k^* = \left[\frac{s}{n+g+\delta}\right]^{1/1-\alpha}$$

$$y = k^{\alpha} \longrightarrow y^* = k^{*\alpha}$$

$$= \left[\frac{s}{n+g+\delta}\right]^{\alpha/1-\alpha}$$

$$c = (1-s)y \longrightarrow c^* = (1-s)y^*$$

$$= (1-s)\left[\frac{s}{n+g+\delta}\right]^{\alpha/1-\alpha}$$

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(b) Golden rule

where c is max

take $k^* = \left[\dfrac{s}{n+g+\delta}\right]^{1/1-\alpha}$

solve for s^* ⟶

$$s^* = (n+g+\delta)\,k^{*1-\alpha}$$

into c^* from (a)

$$c^* = \left[1-(n+g+\delta)k^{*1-\alpha}\right]\left[\frac{(n+g+\delta)k^{*1-\alpha}}{n+g+\delta}\right]^{\frac{\alpha}{1-\alpha}}$$

simplify to

$$c^* = k^{*\alpha} - (n+g+\delta)k^*$$

max it wrt to k

$$\frac{\partial c^*}{\partial k^*} = \alpha k^{*\alpha-1} - (n+g+\delta) = 0$$

$$\boxed{k^*_{GR} = \left[\frac{\alpha}{n+g+\delta}\right]^{1/1-\alpha}}$$

(c)

$$\boxed{s^*_{GR} = \alpha} \Longleftarrow s^*_{GR} = \left[\frac{n+g+\delta}{\alpha/(n+g+\delta)}\right]^{1-\alpha/1-\alpha}$$

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Question 2

(a) $$Y = ALf(k) = ALf\left(\frac{K}{AL}\right)$$

$$w \equiv \frac{\partial Y}{\partial L} = ALf'(k)\left[-\frac{K}{AL^2}\right] + Af(k)$$

$$= A\left[\left(\frac{-K}{AL}\right) f'(k) + f(k)\right]$$

$$= \boxed{A\left[f(k) - kf'(k)\right]}$$

(b) $$Y = ALf(k) = ALf\left(\frac{K}{AL}\right)$$

$$r \equiv \frac{\partial Y}{\partial K} - \delta = ALf'(k)\frac{1}{AL} - \delta = \boxed{f'(k) - \delta}$$

into $wL + rK$

$$wL + rK = A\left[f(k) - kf'(k)\right]L + \left(f'(k) - \delta\right)K$$

simplify

$$= ALf\left(\frac{K}{AL}, 1\right) - \delta K$$

$$= F(K, AL) - \delta K$$

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(c) $r = f'(k) - \delta$

δ: Constant

k^* is Constant

$f'(k^*)$ is Constant

$\longrightarrow \dfrac{\dot r}{r} = 0$

share of Y going to K $= \dfrac{rK}{Y}$

$$\frac{\dot{\left(rK/Y\right)}}{rK/Y} = \frac{\dot r}{r} + \frac{\dot K}{K} - \frac{\dot Y}{Y} = 0 + (n+g) - (n+g) = 0$$

$$w = A\left[f(k) - kf'(k)\right]$$

$$\frac{\dot w}{w} = \frac{\dot A}{A} + \frac{\dot{\left(f(k) - kf'(k)\right)}}{f(k) - kf'(k)} = g + \frac{f'(k)\dot k - \dot k f'(k) - kf''(k)\dot k}{f(k) - kf'(k)}$$

$$= g + \frac{-kf''(k)\dot k}{f(k) - kf'(k)}$$

when $\dot k = 0 \longrightarrow \dfrac{\dot w}{w} = g$

(d) if $k < k^* \longrightarrow \dfrac{\dot w}{w} > g$ b/c $\left[f(k) - kf'(k)\right]$ is +ve

$(kf''(k)\dot k)$ is −ve b/c $f''() < 0$

$$\frac{\dot r}{r} = \frac{\dot{\left[f'(k)\right]}}{f'(k)} = \frac{\overset{-ve}{f''(k)}\,\overset{+ve}{\dot k}}{\underset{+ve}{f'(k)}}$$

as $k \uparrow$ to k^*

$\dot r / r < 0$