College of Business Administration
Department of Economics
Aggregate ECN (Macroeconomics)
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ECO 6206
Fall 2003

Assignment I & Solution

Question 1: Growth Rates

The variables are technology A, output Y and labor L. g_A, g_Y, g_L are the growth rates for each, respectively. Prove the following:

1. If $A = \frac{Y}{L}$ then $g_A = g_Y - g_L$

   `Solution:`

   `Take logs, yileds` $\log A = \log Y - \log L$, `then differentiate with respect to time,` $g_A = d\log A/dt = d\log Y/dt - d\log L/dt = \dot{Y}/Y - \dot{L}/L = g_Y - g_L$

2. If $Y = L^\alpha$ then $g_Y = \alpha g_L$

   `Solution:`

   `Take logs, yileds` $\log Y = \alpha \log L$, `then differentiate with respect to time.`

3. If $Y = \beta L$ then $g_Y = g_L$

   `Solution:`

   `Take logs, yileds` $\log Y = \log \beta + \log L$, `then differentiate with respect to time,` $g_Y = \dot{Y}/Y = \dot{\beta}/\beta + \dot{L}/L = 0 + g_L$.

Question 2: The Harrod-Domar Growth Model

Let's consider a continuous version of the Harrod-Domar growth model. In this model, savings S, is assumed to be proportional to income Y. Investment I (the change in the capital stock) is proportional to the change in income over time. In equilibrium, Investment equals Savings. If s denotes the average (here equal to the marginal) propensity to save, and v the coefficient for the investment relationship, then the model can be captured by the following set of equations,

$$S = sY \tag{1}$$
$$I = \dot{K} = v\dot{Y} \tag{2}$$
$$I = S \tag{3}$$

where a dot above the variable denotes the first time derivative, i.e., dx/dt. Substituting $v\dot{Y} = sY$, we immediately derive the following homogeneous differential equation,

$$\dot{Y} - \left(\frac{s}{v}\right) Y = 0 \tag{4}$$

with initial condition, $I_0 = S_0 = sY_0$.

1. From equation (4), what is the rate of growth of income g_Y?

   `Solution:`

   `The growth of income` $g_Y = \dot{Y}/Y = s/v$

2. What is the solution path for income $Y(t)$? (Solve the differential equation)

   `Solution:`

$$\begin{aligned}
\dot{Y} - \left(\frac{s}{v}\right) Y &= 0 && (5)\\
\dot{Y}\frac{1}{Y} &= \left(\frac{s}{v}\right) && (6)\\
\int \frac{dY}{dt}\frac{1}{Y}dt &= \int \left(\frac{s}{v}\right) dt && (7)\\
\ln Y_t &= \frac{s}{v}t && (8)\\
Y_t &= Y_0 \exp(\frac{s}{v}t) && (9)
\end{aligned}$$