

Assignment I

Question 1: Growth Rates

The variables are technology A , output Y and labor L . g_A, g_Y, g_L are the growth rates for each, respectively. Prove the following:

1. If $A = \frac{Y}{L}$ then $g_A = g_Y - g_L$
2. If $Y = L^\alpha$ then $g_Y = \alpha g_L$
3. If $Y = \beta L$ then $g_Y = g_L$

Question 2: The Harrod-Domar Growth Model

Let's consider a continuous version of the Harrod-Domar growth model. In this model, savings S , is assumed to be proportional to income Y . Investment I (the change in the capital stock) is proportional to the change in income over time. In equilibrium, Investment equals Savings. If s denotes the average (here equal to the marginal) propensity to save, and v the coefficient for the investment relationship, then the model can be captured by the following set of equations,

$$S = sY \quad (1)$$

$$I = \dot{K} = v\dot{Y} \quad (2)$$

$$I = S \quad (3)$$

where a dot above the variable denotes the first time derivative, i.e., dx/dt . Substituting $v\dot{Y} = sY$, we immediately derive the following homogeneous differential equation,

$$\dot{Y} - \left(\frac{s}{v}\right)Y = 0 \quad (4)$$

with initial condition, $I_0 = S_0 = sY_0$.

1. From equation (4), what is the rate of growth of income g_Y ?
2. What is the solution path for income $Y(t)$? (Solve the differential equation)