

Assignment III

Question 1: Neoclassical Growth with Fixed Labor Supply.

Solve the following infinite-horizon continuous-time optimization problem faced by the representative consumer.

$$\max_{c_t} \int_{t=0}^{\infty} e^{-\rho t} \frac{C_t^{1-\theta}}{1-\theta} dt \quad (1)$$

subject to

$$\dot{K}_t = AK_t^\alpha N^{1-\alpha} - C_t - \delta K_t \quad (2)$$

$$K_0 \text{ is given and positive} \quad (3)$$

where C_t, K_t, N denotes consumption, capital and labor, respectively. ρ denotes the rate of time preference. δ denotes the capital depreciation and A is a technology constant. For now, θ is just a utility parameter (later, we will address its properties). Equation (2) is the transition equation for capital.

Solve the problem (get the two differential equations) and graph them in the phase plane for \dot{C}_t and \dot{K}_t .

Question 2: Using the Lagrangian.

Consider the following discrete-time problem faced by the representative consumer,

$$\max_{c_t} \sum_{t=0}^2 \beta^t \ln(c_t) \quad (4)$$

subject to

$$a_{t+1} = (1+r)a_t - c_t \quad t = 0, 1, 2 \quad (5)$$

$$a_0 = \bar{a}_0 \quad \text{Initial Condition} \quad (6)$$

$$a_3 = 0 \quad \text{End Condition} \quad (7)$$

where c_t denotes consumption at time t . The representative consumer lives only for 3 periods ($t = 0, 1, 2$). a_t denotes the assets that the representative consumer have at time t . Equation (5) is the transition equation for assets. It describes how assets evolve over time. β is the subjective probability of living to tomorrow acting as a discount factor. r is the real interest rate.

1. Solve the problem using the Lagrangian method (i.e., find c_0 as function of a_0 , and find c_1 as function of a_1). Note that $c_2 = (1 + r)a_2$ from combining equations (5) and (7).
2. If $\beta = 0.99$ and $r = 4\%$, graph the optimal consumption path (graph c_0, c_1 and c_2 over time.)
3. What will happen to the consumption path if β drops from 0.99 to 0.1? Explain and solve for the new optimal consumption path. Graph the new optimal consumption path using $\beta = 0.1$ (on the same graph of question 2.)