College of Business Administration
Department of Economics
Aggregate ECN (Macroeconomics)
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ECO 6206
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Assignment IV

Question 1: Constant Consumption Growth and Revealed Rate of Time Preference.

Consider a household that maximizes her/his utility subject to the discounted lifetime budget as follows,

$$\max_{C_t} U = \int_{t=0}^{\infty} e^{-\rho t} \frac{C_t^{1-\theta}}{1-\theta} \frac{L_t}{H} dt \tag{1}$$

subject to

$$\int_{t=0}^{\infty} e^{-rt} C_t \frac{L_t}{H} dt = W \tag{2}$$

Note the following: (a) the real interest rate r is constant, and (b) W denotes the initial wealth and the present value of the lifetime labor income.
The problem is basically the same as in Chapter 2 [equations (2.1) and (2.2)], except for that what is related to r and W.

1. Find the utility-maximizing path of C, given r, W, and the parameters of the utility function. (Solve and find C_t)

2. Show that the consumption growth rate ($\dot{C}_t/C_t$) is constant.

3. Show that if $r > \rho$, then consumption will be rising over time (i.e., if a rising consumption is observed, then it must be that the market real interest rate is higher than the rate of time preference, hence the title of the question "revealed rate of time preference".)

Question 2: The CRRA Momentary Utility and θ.

Given the constant relative risk aversion CRRA utility,

$$U(C_t) = \frac{C_t^{1-\theta} - 1}{1-\theta} \tag{3}$$

Using Excel (or any other software), graph the CRRA utility for the following values of $\theta = \{-0.9, -0.5, 0.1, 0.5, 1\}$. Put all the utility functions on the same graph. The point of this exercise is to illustrate the shape of the utility under different values of θ. Explain the shape of the utility as function of θ.