College of Business Administration
Department of Economics
Aggregate ECN (Macroeconomics)
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Assignment V

Question 1: The Hodrick-Prescott (HP) Filter.

The first step of the HP curve-fitting method is to take the logarithms of the variables for two reasons: 1) to compress the units in which the variables are measured in, and 2) because of the inherent exponential trend in most aggregate economic variables. The selected trend path $\{\tau_t\}$ is one which minimizes the sum of squared deviations from a given series $\{Y_t\}$ subject to the constraint that the sum of the squared sum differences not be large. Formally, the minimization problem reduces to

$$\min_{\{\tau_t\}_{t=1}^{T}} \sum_{t=1}^{T}(Y_t - \tau_t)^2 + \lambda \cdot \sum_{t=2}^{T-1}\left[(\tau_{t+1} - \tau_t) - (\tau_t - \tau_{t-1})\right]^2 \tag{1}$$

The second sum of the squared term is an approximation of the derivative of τ_t at time t. One attempts to minimize two sums of squares: the sum of squared cyclical residuals and the sum of squared $\Delta^2\tau_t$. The smoothing parameter λ gives relative weight to these two sums of squares. For annual data, λ is set equal to 100; for monthly data, λ is set equal to 14,400 and for quarterly data, λ is set equal to 1600[1]. This parameter acts as a penalty for the acceleration of growth. Finally, the cyclical component of the time series is computed as a deviation from the trend,

$$Y_t^c = Y_t - \tau_t \qquad \text{for} \quad t = 1, ...T \tag{2}$$

Using Excel, reproduce Table 1 (Cyclical Behavior of the U.S. Economy) as reported by Edward Prescott (1986) "Theory Ahead of Business Cycle Measurement" Federal Reserve Bank of Minneapolis, Quarterly Review Vol. 10 No. 4 (A link to the paper is available at the class web page). To do so, follow the next steps:

1. Download the data from the Federal Reserve Bank of St. Louis,
   <http://research.stlouisfed.org/fred2/>

[1]The rationale for setting $\lambda = 1600$ is as follows. The parameter $\lambda = \sigma_c^2/\sigma_\tau^2$, where σ_c^2 denotes the variance of the cyclical component and σ_τ^2 denotes the variance of the trend component. Hodrick and Prescott used "... the prior view that a five percent cyclical component is moderately as large as is one-eight of one percent change in the rate of growth in a quarter ...". Therefore, $\lambda^{1/2} = \frac{5/1}{1/8}$ or $\lambda = 1600$ as a value for the smoothing parameter.

2. Download the Statistics Package from (this package contains the HP filter), *<http://www.spinnakeradd-ins.com/spinnaker_stats.htm>*

3. Install the Statistics Package as an Add-Ins in Excel. (using *Tools*, *Add-Ins* and then *Browse*)

4. Take the log of each time series.

5. Create the trend by HP filtering each time series.

6. Compute the cyclical component of each series by taking the series from step (4) minus the series from step (5).

7. Using the time series from step (6), compute the standard deviation of each series and the correlation with GDP.

8. Graph the cyclical component of output.

Question 2: A Simplified RBC with Additive Shocks.

Following Blanchard and Fischer (1989, MIT Press, pp. 329-331), consider an economy consisting of a constant population of infinitely lived individuals. The representative individual maximizes the expected value of $\sum_{t=0}^{\infty} \beta^t u(c_t)$. The momentary utility function is quadratic and defined as $u(c_t) = c_t - \theta c_t^2$ where $\theta > 0$. Assume that c_t is is always in the range where $u^{'}(c_t)$ is positive. β is the subjective discount rate and it is defined as $\beta \equiv 1/(1+\rho)$, where ρ is the rate of time preference.
Output is linear in capital, plus an additive disturbance: $Y_t = AK_t + e_t$, where $e_t = \phi e_{t-1} + \varepsilon_t$, where $-1 < \phi < 1$ and where the ε_t 's are mean-zero, i.i.d. shocks. There is no depreciation; thus $K_{t+1} = K_t + Y_t - C_t$, and the interest rate (r) is constant. Assume that $r = \rho$.

Find the first-order condition (Euler Equation) relating c_t to the expectation of c_{t+1}.