

Assignment

Diamond Overlapping Generations (OLG)

Consider the Diamond overlapping-generations model with population growth and no technology growth. Assume that there is a large number of identical firms. The number of (identical) people born in period t is denoted by N_t and is given by

$$N_{t+1} = (1 - \eta)N_t \quad \eta > 0$$

Aggregate output in this economy in period t is denoted by Y_t and given by the Cobb-Douglas production function

$$Y_t = K_t^\alpha N_t^{1-\alpha} \quad 0 < \alpha < 1$$

where K_t denotes the aggregate capital stock. Profit maximization by firms implies that the return on capital (real interest rate r_t) in period t equals to the marginal product of capital and that the real wage rate w_t equals to the marginal product of labor.

Assume that individuals live for two periods. An individual is endowed with one unit of labor time in the first period of life (youth) and zero units of labor time in the second period of life (old age). All young individuals supply labor inelastically and invest their savings in capital. A member of generation t maximizes lifetime utility

$$U(c_{1t}, c_{2t+1}) = \ln c_{1t} + \beta \ln c_{2t+1} \quad 0 < \beta < 1$$

subject to the budget constraints in the two periods,

$$\begin{aligned} c_{1t} &= w_t - s_t \\ c_{2t+1} &= (1 + r_{t+1})s_t \end{aligned}$$

where w_t refers to the wage income and s_t denotes the amount of savings.

1. Derive the Aggregate Saving S_t for this economy. Discuss.
2. Let $k_t \equiv K_t/N_t$. Derive a nonlinear first-difference equation describing the evolution of the capital-labor ratio in the competitive equilibrium economy.
3. Let k^* denotes the steady-state value of the capital-labor ratio. Solve for k^* using the equation you derived in part (2.). Also, show k^* on a phase diagram where k_{t+1} is measured on the vertical axis and k_t is on the horizontal axis.
4. Assume that the economy is on its balanced growth path. Derive the growth rates of k , K and Y . Interpret your results and compare them to the results of the standard Solow growth model.
5. Assume that the population growth rate (η) drops. Using the phase diagram from (3.), discuss the changes that will take place in this economy.