NAME: ______________________________

College of Business Administration
Department of Economics
Aggregate Economic Conditions
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ECO 6206
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Final Exam

- This is a closed book exam.
- Time: 6:00 p.m. to 8:45 p.m. Location: BA2-301.
- The exam totals 250 points.
- Answer all five questions.
- The questions are equally distributed. Each question is worth 50 points.
- Answer in the given space after each question.
- Write your name on the exam booklet.
- Calculators and language dictionary are allowed.
- EXPLAIN all your end-result derivations.
- Write CLEARLY, CLEARLY, CLEARLY.

Question I

The Dynamics of Growth

Consider the following economy where a central planner seeks to maximize the representative consumer utility

$$\sum_{t=0}^{\infty} \beta^t \ln c_t \qquad\qquad 0 < \beta < 1$$

subject to the resource constraint

$$c_t + k_{t+1} = Ak_t$$

Where A is a positive constant.

1. What is (are) the state variable(s) and what are the control (choice) variables for this problem?
2. Solve for the optimal consumption path.
3. What is the growth rate of c_t?
4. Describe the dynamics of the model when $A\beta > 1$, when $A\beta < 1$ and when $A\beta = 1$.
5. What is the factor explaining growth in this model?
6. Solve for Investment (I_t).
7. What is the value of the capital depreciation rate.
8. Do we need the central planner to solve this problem? How else can we solve this problem? why? Explain.

Question II

The Benchmark Real Business Cycle model

Consider the following economy, wherein the representative household

$$\max E_t \sum_{t=0}^{\infty} \beta^t \left[\ln c_t + b \frac{(1-l_t)^{1-\gamma}}{1-\gamma} \right] \qquad b > 0, \qquad \gamma > 0 \tag{1}$$

subject to,

$$y_t = k_t^{\alpha}(A_t l_t)^{1-\alpha} \tag{2}$$

$$k_{t+1} = k_t + I_t - \delta k_t \tag{3}$$

$$A_t = \rho A_{t-1} + \epsilon_t \tag{4}$$

where y_t, c_t, k_t, I_t, l_t refer to output, consumption, capital, investment and labor, respectively. A_t refers to technology shocks and ϵ_t are white noise disturbances. E_t is the expectation operator.

1. Explain what is the parameter b? If $b = 2/3$, what does this mean?
2. Explain what is the parameter γ? If $\gamma = 1$, write the expression for the momentary utility.
3. Explain in words what are equations (2), (3) and (4).
4. The model is missing one equation, write the missing equation to make the model complete.
5. Find the first-order condition that relates current leisure and consumption, given the wage.
6. Is leisure $(1 - l)$ constant?
7. Why is this model referred to as a 'Real' Business Cycle?

Question III

Money Demand, the Real Disposable Income in a Closed and Open Economy

Suppose that the demand for real money balances depends on the interest rate, i, and on *disposable* income $Y - T$; in other words, suppose that the correct way to write the LM equation is $M/P = L(i, Y - T)$.

1. With this change to the IS-LM-AS model, can one tell whether a tax cut (that is, a fall in T) increases or decreases output? Assume a closed economy. Assume that P and G are held constants. Discuss your results. Supplement your answer by a graph.

2. Redo part (1) assuming an open economy under the assumptions that the exchange rate is floating, exchange-rate expectations are static, and capital is perfectly mobile. Discuss your results. Supplement your answer by a graph.

3. Redo part (2) assuming a fixed exchange rate. Discuss your results. Supplement your answer by a graph.

Question IV

Consumption Function and Endogenous Savings

Consider an economy with overlapping generations where 200 identical individuals are born in each period (i.e., $N_t = 200$ for $t = 0, 1, 2, ...$) In this economy, there exists a private borrowing/lending market in which agents can participate. An individual born in period t maximizes her utility subject to her budget,

$$u(c_{1t}, c_{2t+1}) = \ln c_{1t} + 0.8 \ln c_{2t+1}$$

subject to

$$c_{1t} + \frac{c_{2t+1}}{1 + r_t} = y_{1t} + \frac{y_{2t+1}}{1 + r_t}$$

An individual born in period t has the following endowment stream $\{y_{1t}, y_{2t+1}\} = \{2, 1.76\}$.

1. Derive the Euler equation of the individual born in period t.
2. Derive the consumption function in young age as function of the interest rate, i.e., derive $c_{1t}(r_t)$.
3. Derive the saving function in young age as function of the interest rate, i.e., derive $s_t(r_t)$.
4. Solve for the numerical value of the equilibrium interest rate r_t in period t.
5. What are the sequence of equilibrium interest rate and the consumption allocation in the economy?

Question V

Log-Linear Demand, Taste Shocks with Lucas Perfect/Imperfect Information

Suppose that the consumption index C_i is defines as $C_i = \left[\int_{j=0}^{1} Z_j^{1/\eta} C_{ij}^{(\eta-1)/\eta} dj\right]^{\eta/(\eta-1)}$, where C_{ij} is the individual's consumption of good j and Z_j is the taste shock for good j. Suppose the individual has amount Y_i to spend on goods. Thus the budget constraint is $\int_{j=0}^{1} P_j C_{ij} dj = Y_i$

1. Find the first-order condition for the problem of maximizing C_i subject to the budget constraint. Solve for C_{ij} in terms of Z_j, P_j and the Lagrange multiplier on the budget constraint.
2. Use the budget constraint to find C_{ij} in terms of Z_j, P_j, Y_i, and the $Z's$ and $P's$.
3. Substitute your results in part (2) into the expression for C_i and show that $C_i = Y_i/P$ where $P = \left(\int_{j=0}^{1} Z_j P^{1-\eta} dj\right)^{1/(1-\eta)}$
4. Use your results in part (2), and part (3) to show that $C_{ij} = Z_j(P_j/P)^{-\eta}(Y_i/P)$.
5. Compare your results with the demand for good i, $c_i = y + z_i - \eta(p_i - p)$ and $p = \overline{p_i}$.
6. Explain in details and in simple terms what is C_i?
7. What does j refer to? What does i refer to?