NAME: ______________________________

College of Business Administration
Department of Economics
Aggregate Economic Conditions
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ECO 6206
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Mid-Term Exam

- This is a closed book exam.
- The exam totals 120 points.
- Time: 6:00 p.m. to 8:45 p.m.
- Write your name on the exam booklet.
- Calculators and language dictionary are allowed.
- Answer all the questions.
- Answer in the given space after each question.
- EXPLAIN all your end-result derivations.
- Write CLEARLY, CLEARLY, CLEARLY.

Question I:
Intertemporal Substitution in Labor Supply (20 points)

To see what the utility function implies for labor supply, consider a household that lives only for one period and has no initial wealth. In addition, assume for simplicity that the household has only one member. In this case, the household's problem is to maximize the momentary utility $U(c, 1-l) = \ln c + b\ln(1-l)$ subject to the budget constraint $c = wl$. Where c and l refer to consumption and labor. w denotes the real wage.

1. What does b refer to? Explain.
2. Write the Lagrangian for the household's problem.
3. Derive the first order conditions and show that the income and substitution effects of a change in the wage offset each other.
4. Now, assume that the household lives for two periods. Write the problem that she faces.
5. Using (4.) Write the Lagrangian.
6. Using (5.) Derive the first order conditions.
7. The effects of a change in the real interest rate r on the labor supply is a crucial argument for the Real Business Cycle (RBC) models. Using (6.), show and discuss the intertemporal substitution in labor supply.

Question II:

Cake Eating in Discrete and Continuous Time (20 points)

Once upon a time there was a little girl who got *one* cake. The girl decided to eat the cake alone. But she was undetermined *when* she wanted to it. First, she thought of eating the whole cake right away. But then, nothing would be left for tomorrow and the day after tomorrow. Well, on one hand, she thought by herself, eating cake today is better than eating it tomorrow. On the other hand, eating too much at the same time might not be the best thing to do. She imagined that the first mouthful of cake is a real treat, the second is great, the third is also nice. But the more you eat, the less you enjoy it. In the end, you're almost indifferent, she thought. So, she decided to eat only a bit of the cake everyday. Then, she could eat everyday another first mouthful of cake. The *rational* girl is not impatient. The girl knew that the cake would be spoiled if she kept it more than nine days. Therefore, she would eat the cake in the first ten days. Yet, how much should she eat everyday? Find the optimal consumption path using Discrete time and Continuous time. Note that the problem is deterministic.

1. Write the maximization problem in Discrete Time.
2. Solve for the optimal consumption path using the Lagrangian.
3. Write the same maximization problem in Continuous Time.
4. Solve for the optimal consumption path using the Hamiltonian.

Question III:
Diamond Overlapping Generations (OLG) (40 points)

Consider the Diamond overlapping-generations model with population growth and no technology growth. Assume that there is a large number of identical firms. The number of (identical) people born in period t is denoted by N_t and is given by

$$N_{t+1} = (1 - \eta)N_t \qquad\qquad \eta > 0$$

Aggregate output in this economy in period t is denoted by Y_t and given by the Cobb-Douglas production function

$$Y_t = K_t^{\alpha} N_t^{1-\alpha} \qquad\qquad 0 < \alpha < 1$$

where K_t denotes the aggregate capital stock. Profit maximization by firms implies that the return on capital (real interest rate r_t) in period t equals to the marginal product of capital and that the real wage rate w_t equals to the marginal product of labor.

Assume that individuals live for two periods. An individual is endowed with one unit of labor time in the first period of life (youth) and zero units of labor time in the second period of life (old age). All young individuals supply labor inelastically and invest their savings in capital. A member of generation t maximizes lifetime utility

$$U(c_{1t}, c_{2t+1}) = \ln c_{1t} + \beta \ln c_{2t+1} \qquad\qquad 0 < \beta < 1$$

subject to the budget constraints in the two periods,

$$\begin{aligned} c_{1t} &= w_t - s_t \\ c_{2t+1} &= (1 + r_{t+1})s_t \end{aligned}$$

where w_t refers to the wage income and s_t denotes the amount of savings.

1. Derive the Aggregate Saving S_t for this economy. Discuss.
2. Let $k_t \equiv K_t/N_t$. Derive a nonlinear first-difference equation describing the evolution of the capital-labor ratio in the competitive equilibrium economy.
3. Let k^* denotes the steady-state value of the capital-labor ratio. Solve for k^* using the equation you derived in part (2.). Also, show k^* on a phase diagram where k_{t+1} is measured on the vertical axis and k_t is on the horizontal axis.
4. Assume that the economy is on its balanced growth path. Derive the growth rates of k, K and Y. Interpret your results and compare them to the results of the standard Solow growth model.
5. Assume that the population growth rate (η) drops. Using the phase diagram from (3.), discuss the changes that will take place in this economy.

Continue Answer Question III

Question IV:
Consumption Function and Endogenous Savings (20 points)

Consider an economy with overlapping generations where 200 identical individuals are born in each period (i.e., $N_t = 200$ for $t = 0, 1, 2, ...$) In this economy, there exists a private borrowing/lending market in which agents can participate. An individual born in period t maximizes her utility subject to her budget,

$$u(c_{1t}, c_{2t+1}) = \ln c_{1t} + 0.8 \ln c_{2t+1}$$

subject to

$$c_{1t} + \frac{c_{2t+1}}{1 + r_t} = y_{1t} + \frac{y_{2t+1}}{1 + r_t}$$

An individual born in period t has the following endowment stream $\{y_{1t}, y_{2t+1}\} = \{2, 1.76\}$.

1. Derive the Euler equation of the individual born in period t.
2. Derive the consumption function in young age as function of the interest rate, i.e., derive $c_{1t}(r_t)$.
3. Derive the saving function in young age as function of the interest rate, i.e., derive $s_t(r_t)$.
4. Solve for the numerical value of the equilibrium interest rate r_t in period t.
5. What are the sequence of equilibrium interest rate and the consumption allocation in the economy?

Question V:
The Liquidity Trap and the Pigou Effect (20 points).

Assume that the nominal interest rate is so low that the opportunity cost of holding money is negligible. Suppose that the as a result people are indifferent concerning the division of their wealth between money and other assets, and that they are therefore willing to change their money holdings without any change in the interest rate.

1. The liquidity trap (Keynes 1936).
   In this situation, what is the slope of the AD curve? If prices are completely flexible, is aggregate demand irrelevant to output?

2. The Pigou effect (Pigou 1943).
   Sine the public's holdings of high-powered money (M/P) are one component of wealth, a fall in the price level increases real wealth. Suppose that, in addition, planned expenditure depends on real wealth as well as the variables in $E = E(Y, i - \pi^e, G, T, M/P)$, and $0 < E_Y < 1$, $E_{i-\pi^e} < 0$, $E_G > 0$, and $E_T < 0$. If prices are completely flexible, is aggregate demand irrelevant to output?