The Standard Real Business Cycle Model with Environmental Quality

Social Planner Problem

Maximise
$$E\left[\sum_{t=0}^{\infty} \beta^{t}\cdot\left(\log(c_t)+\log(E_t)\right)\right]$$

Subject to
$$C_t+i_t+m_t = A\cdot k^{\alpha}\cdot h^{1-\alpha}$$

$$E_{t+1} = (1-\delta_E)\cdot E_t+z_t\cdot m_t-\phi\cdot c_t$$

$$k_{t+1} = (1-\delta_k)\cdot k_t+i_t$$

$$\ln(z_{t+1}) = \rho\cdot\ln(z_t)+\ln(\varepsilon_t)$$

The Social Planner

$$E\left[\sum_{t=0}^{\infty} \beta^{t}\cdot\left[\begin{array}{l}\left(\log(c_t)+\log(E_t)\right)\;\ldots\\ +\lambda\cdot\left[c_t+k_{t+1}-(1-\delta_k)\cdot k_t+\dfrac{\left[E_{t+1}-(1-\delta_E)\cdot E_t+\phi\cdot c_t\right]}{z_t}-A\cdot k^{\alpha}\cdot h^{1-\alpha}\right]\end{array}\right]\right]$$

First Order Conditions

$$\frac{1}{c_t}-\lambda-\lambda\cdot\frac{\phi}{z_t} = 0$$

$$-\lambda+\beta\cdot E_0\cdot\lambda\cdot\left[(1-\delta_k)+\alpha\cdot A\cdot k^{\alpha-1}\right] = 0$$

$$\frac{-\lambda}{z_t}+\beta\cdot\frac{1}{E_{t+1}}+\beta\cdot\lambda\cdot\frac{(1-\delta_E)}{z_t} = 0$$

The steady State

$$\lambda = \frac{1}{c \cdot \left(1 + \frac{\phi}{z}\right)}$$

$$k = \left[\frac{\beta \cdot \alpha \cdot A}{1 - \beta \cdot (1 - \delta k)}\right]^{\frac{1}{1-\alpha}}$$

$$E = \frac{\beta \cdot z}{\lambda \cdot [1 - \beta \cdot (1 - \delta E)]}$$

$$m = \frac{E - (1 - \delta E) \cdot E + \phi \cdot c}{z}$$

$$i = k - (1 - \delta k) \cdot k$$

$$y = A \cdot k^{\alpha}$$

$$c = y - i - m$$

Solving for the Steady State

Calibrate the parameters.

$\alpha := 0.35$	Capital's Share of Output	$\delta E := 0.1$	Depreciation Rate for Environment
$\beta := 0.96$	Discount Rate	$\delta k := 0.06$	Depreciation Rate for Capital
$\phi := 0.1$	degradation due to cons'	$z := 2$	Steady State value for the Log-Normal shock

Initializations

$k := 10 \quad A := 1 \quad E := 1 \quad i := 1 \quad c := 1 \quad m := 1 \quad y := 1$

Given

$$A = \frac{1 - \beta \cdot (1 - \delta k)}{\beta \cdot \alpha \cdot k^{\alpha - 1}}$$

$$E = \frac{\beta \cdot z}{\frac{1}{c \cdot \left(1 + \frac{\phi}{z}\right)} \cdot \left[1 - \beta \cdot (1 - \delta E)\right]}$$

$$i = k - (1 - \delta k) \cdot k$$

$$y = A \cdot k^{\alpha}$$

$$m = \frac{E - (1 - \delta E) \cdot E + \phi \cdot (y - i - m)}{z}$$

$$c = y - i - m$$

$$g1(\alpha, \beta, \delta E, \delta k, z, \phi, k) := \mathrm{Find}(A, E, c, m, i, y)$$

$$g1(\alpha, \beta, \delta E, \delta k, z, \phi, k) = \begin{pmatrix} 1.298 \\ 19.074 \\ 1.287 \\ 1.018 \\ 0.6 \\ 2.905 \end{pmatrix}$$

Analysis at the Steady State

We graph the steady state effects on consumption c, Environmental Quality E, Environmental Investment m, investment i and output y.

The variables are functions of the parameters.

$$\mathrm{Environment}(\alpha, \beta, \delta E, \delta k, z, \phi, k) := g1(\alpha, \beta, \delta E, \delta k, z, \phi, k)_1 \quad \mathrm{Environment}(\alpha, \beta, \delta E, \delta k, z, \phi, k) = 19.074$$

$$\mathrm{Consumption}(\alpha, \beta, \delta E, \delta k, z, \phi, k) := g1(\alpha, \beta, \delta E, \delta k, z, \phi, k)_2 \quad \mathrm{Consumption}(\alpha, \beta, \delta E, \delta k, z, \phi, k) = 1.287$$

$$\mathrm{EnvInvest}(\alpha, \beta, \delta E, \delta k, z, \phi, k) := g1(\alpha, \beta, \delta E, \delta k, z, \phi, k)_3 \quad \mathrm{EnvInvest}(\alpha, \beta, \delta E, \delta k, z, \phi, k) = 1.018$$

$$\text{Investment}(\alpha,\beta,\delta E,\delta k,z,\phi,k) := g1(\alpha,\beta,\delta E,\delta k,z,\phi,k)_4 \qquad \text{Investment}(\alpha,\beta,\delta E,\delta k,z,\phi,k) = 0.6$$

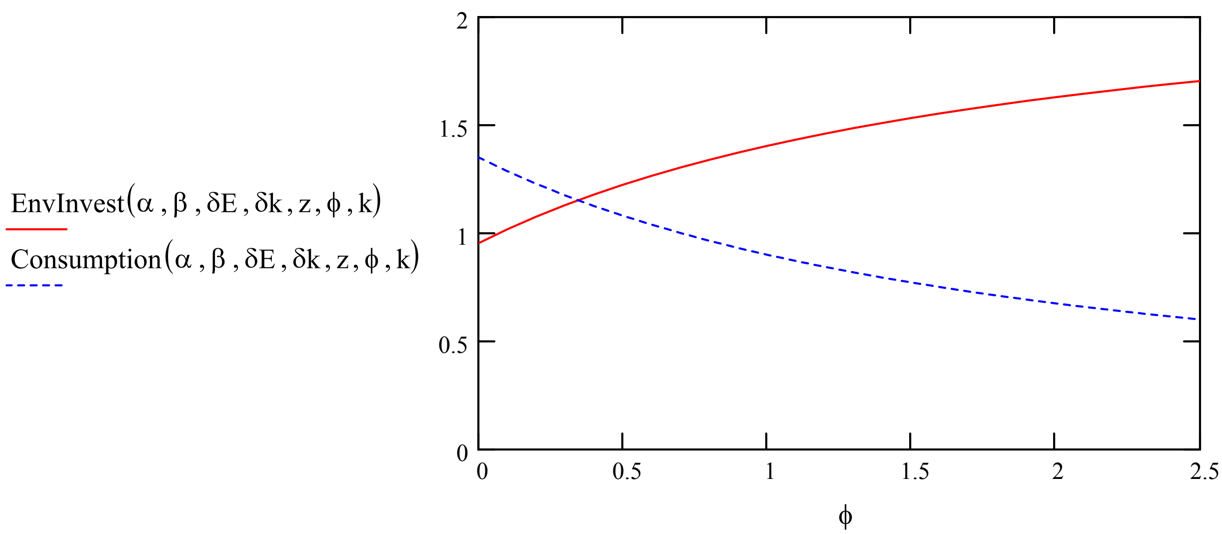
$$\text{Output}(\alpha,\beta,\delta E,\delta k,z,\phi,k) := g1(\alpha,\beta,\delta E,\delta k,z,\phi,k)_5 \qquad \text{Output}(\alpha,\beta,\delta E,\delta k,z,\phi,k) = 2.905$$

Creating a range variable.

$\phi := 0.0, 0.1 .. 2.5$ Change it for own interest

If you change the x-axis label. The graph will then be redrawn for the new range

Graph of the effects of degradation on Consumption and the Investment in Environmental Quality

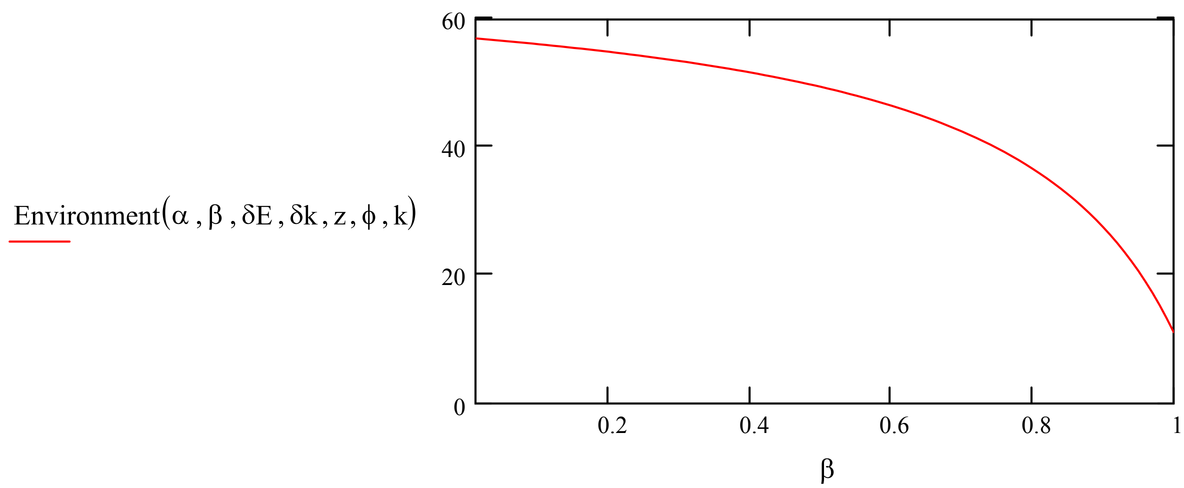


As ϕ increases, and given the log nature of the Utility which implies perfect smoothing, consumption decrases and investment in E quality increases proportionaly.

$\phi := 0.1$

$\beta := 0.01, 0.02 .. 1$

Graph of Effect of the Discount Rate on Environmental Quality



As the cost of capital (interest rate) increases (i.e., β decreases), the representative household substitute away from the expensive good (capital) towards the cheaper good (environmental quality)

$\beta := 0.96$

$\delta E := 0.01, 0.02 .. 1$

Graph the effect of the depreciation rate for E Quality on Consumption and Investment in E Quality