Beyond Backyard Environmentalism: A Stochastic Dynamic General Equilibrium Approach to Environmental Quality.

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February 18, 2002

Abstract

This paper extends the real business cycle framework to incorporate environmental quality. The model is solved using linear quadratic dynamic programming by iterations on the algebraic Ricatti equation. Results allow us to examine the normative desirability of the rolling-rule regime in terms of welfare costs associated with the variability of environmental quality. The rolling-rule regime was proposed in the literature as an evolution to "beyond backyard environmentalism" and as an alternative to existing environmental regulations.

JEL CLASSIFICATION: E32, Q21, Q28.

1 Introduction

The United States is in the midst of an overhaul of its environmental regulation. In general, the aim of the new regulations is to improve environmental quality. Originally grown out of the "Not In My Backyard (NIMBY)" movement[1]; the new set of regulations provide a wider role for environmental activism and lesser variability in environmental regulations' policies.

Beyond the shadow of NIMBY, many communities are organizing to reclaim authority over regional and national environmental issues. Adapting to a new political framework, activists are facing new tasks to determine what should occur in

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their backyard and beyond. Sabel, Fung and Karkkainen (1999) argued that "a new form of decentralized but coordinated environmental regulation is successfully addressing a surprisingly broad range of apparently intractable problems. Even more improbably, its success depends on a new and potentially wide-ranging form of directly deliberative democratic participation." They proposed a new framework to replace the existing central command system with a combination of local experimentation and centralized pooling of experience. The suggested architecture[2] of the "rolling-rule regime" emphasizes accountability of the agents and regulators to each other and to the general public. Most importantly, it advocates the harmonization of regulation' policies. Within this regime, regulators use discretion and available information to periodically reformulate minimum performance standards, desirable targets and paths for the improvement of environmental quality. If adopted, this regime will lower the variability of benchmark standards against which policies and environmental quality are judged.

Critics to the proposed rolling-rule regime express doubts regarding its feasibility and its normative desirability. Among many,[3] a few questioned "how one could know that backyard environmentalism, even if fully realized, would more accurately reflect the desires of citizens, or better translate those desires into effective policy than the current regime."

Regarding feasibility, we adopt the views of John (1999) and Fiorino (1999) who provided examples wherein aspects of the rolling-rule regime were demonstrated in practice and were successful. We do not address this issue.[4]

To investigate its normative desirability, we propose to integrate a few aspects of the rolling-rule regime into a stochastic dynamic general equilibrium model to assess the implications of such a framework on welfare. To emphasize accountability of the agents, we formulate a model wherein the representative agent and the regulator are identical. A representative agent chooses the magnitude of capital and environmental quality to smooth her consumption path. We view this strategy as an appropriate description to address the desirability issue. Other aspects, such as the update of available information to agents, is not addressed for simplicity. In this setup, agents and regulators act in perfect harmony in a dynamic optimization exercise.

Under traditional economic models of individual behavior, household behavior is seen as acting in own-self interest. Any kind of choice and/or trade must improve own welfare. Seldom emphasized in traditional Real Business Cycle models is household' interest in environment quality. We propose integrating environment quality into a Stochastic Dynamic General Equilibrium framework to address the normative desirability of the rolling-rule regime. For a few exceptions wherein environmental quality or pollution was present in a general equilibrium framework, see Forster (1973), John and Pecchenino (1994) and Selden and Song (1995).

In brief, we attempt to theoretically advance the framework of environmentally oriented SDGE models. Specifically, we will investigate the welfare cost to environment quality' fluctuations due to government regulations. Presently, the bulk of research in general equilibrium environment modeling is to rely on growth models as the basis for policy analysis. However, a shortcoming - among many - of such a strategy is that one ignores short-term activism's influence on regulators. Simply put, many short-run implications are not discussed. For example, the implications of government fiscal policies and benchmark standards on environmental quality are usually addressed only in the long run.

There exists no consensus in the literature regarding one important aspect of this research; namely the definition of environmental quality. The lack of a single definition does not imply a lack of interest. Environmental quality has many dimensions and it is a relatively tedious task to assess all. In our model, we include an index for all of the dimensions of environmental quality for which actual measurements exist. We assume that a composite of currently available indicators in the Global Environmental Monitoring System (GEMS) panel data is available. We adopt the Grossman and Krueger' (1995) definition of environmental quality which includes: air and water quality, nature and species diversity. We also suggest adding the quality of soil and groundwater to the definition (John and Pecchenino (1994, p. 1395)).

Section 2 presents the model and its results. Section 3 discusses the associated welfare costs with the variability of investment in environmental quality. Section 4 contains local sensitivity analyses. Section 5 concludes and proposes related future research.

2 The Model

The model that we propose is a derivation from John and Pecchenino (1994, p. 1405). We adapt it to answer the question of normative desirability of the rolling-rule regime. It is a simple stochastic dynamic model in which the representative agent accumulates both environmental quality and capital. Let E_t denotes a broadly defined index for environment quality. One plausible interpretation of E_t is a weighted index of: the quality of soil and groundwater; the cleanliness of rivers and oceans; an index of biodiversity; an inverse of the atmospheric concentration of chlorofluorocarbons, greenhouse gases, and other pollutants.

$$\max_{c_t, E_{t+1}, k_{t+1}} \sum_{t=0}^{\infty} \beta^t U(c_t, E_t) \tag{1}$$

subject to

$$\begin{aligned}
Y_t &= A K_t^{\alpha} N_t^{1-\alpha} && && (2)\\
E_{t+1} &= (1-\delta_E)E_t - \phi c_t + z_t m_t && 0 \le b,\ \delta_E,\ \phi \le 1 && (3)\\
K_{t+1} &= (1-\delta_K)K_t + I_t && 0 \le \delta_K \le 1 && (4)\\
z_{t+1} &= z_t^{\rho} \varepsilon_{t+1} && 0 \le \rho \le 1, \quad \ln(\varepsilon_t) \backsim iid \quad N(\mu,\ \sigma^2) && (5)\\
Y_t &\ge C_t + I_t + m_t && && (6)\\
K_0 &> 0 \qquad m_0 > 0 && && (7)
\end{aligned}$$

Since we regard E_t as a weighted index, it can take only positive values. Therefore, there is no special significance attached to $U(.,0)$. δ_E and δ_K refer to the depreciation rates of environmental quality and capital, respectively. The term ϕc_t refers to the degradation of environment due to consumption. m_t denotes environmental improvement/maintenance. Also, one can refer to m_t as pollution abatement. Note that if $\phi c_t = 0$, then $z_t m_t$ can be interpreted as gross investment in environmental quality. E_t could refer to national parks, which require maintenance. $U(.,.) \in C^2$, $U_c > 0$, $U_E > 0$, $U_{cc} < 0$, $U_{EE} < 0$ and $U_{cE} \ge 0$. Production exhibits constant returns to scale and satisfies the Inada conditions. Note that the stochastic process z_t influences the environmental maintenance m_t to capture regulations' variability. Our innovation is in the way the random shock is modeled. The magnitude of z_t represents the degree of regulators' activism.[5]

The single consumer is assumed to be representative[6] of the society as a whole. A change in the level of her utility reflects and is equivalent to a change in the overall level of social welfare. That is, an increase (decrease) in her utility[7] implies an improvement (loss) in social welfare. The use of the representative agent framework allows us to ignore corner solutions and to treat the participation decision as a continuous variable. Assuming an interior solution guarantees that the representative agent will engage in maintenance. Here, the household is a "dynasty" that derives utility from environmental quality.[8] Given the problem at hand and the emphasis on the rolling-rule regime, the fallacy of composition is avoided. Also note that this framework validates the Samuelson condition for the optimal provision of a public good.[9]

Representative agents' preferences are represented by a utility function which is time separable and state independent. The general momentary utility function is described in the appendix. Here, we adopt the specific form wherein $\sigma = \sigma_1 = 1$. This log-linear utility implies an intertemporal elasticity of substitution of environmental quality equal to one. Given local nonsaturation and no externalities, competitive equilibria - which exist for this l_∞ commodity[10] space economy (Bewley (1972) theorems) - are Pareto Optima (using the competitive welfare theorems of Debreu (1954)). Given a single agent in this economy and convexity, there is a unique optimum to this maximization problem. This optimum is the unique competitive equilibrium allocation and supports the Pareto optimum. Therefore, one can solve the social planner's problem using concave programming techniques. Here, we transform this model into a stochastic linear regulator problem and solve it by iterations on the Ricatti equation.

The social planner solves the following Lagrangian problem,

$$L = \max_{c_t, k_{t+1}, E_{t+1}} E_0 \sum_{t=0}^{\infty} \beta^t \left[\begin{array}{c} \log c_t + \log E_t \\ -\lambda \left(c_t + k_{t+1} - (1-\delta_k)k_t + E_t - (1-\delta_E)E_t + \phi c_t - Ak^a N^{1-\alpha} \right) \end{array} \right] \tag{8}$$

The FOC are:

$$\begin{aligned} c_t &: \quad \frac{1}{c_t} - \lambda(1-\phi) = 0 && (9) \\ k_{t+1} &: \quad -\lambda + \beta E_0 \lambda \left[(1-\delta_k) + \alpha A k^{\alpha-1} \right] = 0 && (10) \\ E_{t+1} &: \quad -\lambda + \beta \frac{1}{E_{t+1}} + \beta\lambda(1-\delta_E) = 0 && (11) \\ \lambda &: \quad c_t + k_{t+1} - (1-\delta_k)k_t + E_t - (1-\delta_E)E_t + \phi c_t - Ak^a N^{1-\alpha} = 0 && (12) \end{aligned}$$

The steady state is computed as,

$$\begin{aligned} \overline{k} &= \left[\frac{\beta \alpha A}{1-\beta(1-\delta_k)} \right]^{\frac{1}{1-\alpha}} && (13) \\ \overline{E} &= \frac{(A\overline{k}^{\alpha} - \delta_k \overline{k})\beta}{\delta_E(1-\beta(1-\delta_E))} && (14) \\ \overline{c} &= \frac{1-\beta(1-\delta_E)}{\beta(1+\phi)} \overline{E} && (15) \\ \overline{i} &= \delta_k \overline{k} && (16) \\ \overline{m} &= \delta_E \overline{E} + \phi \overline{c} && (17) \\ \overline{y} &= A\overline{k}^{\alpha} && (18) \end{aligned}$$

Note that equations (9) and (11) are the dynamic analog of the Samuelson condition for the optimal provision of a public good.

We view variations in environmental improvement as a sign of weakness that originate from different policies applied by different regulators. In the model, z_t captures the stochastic shock to the investment in environmental maintenance. The random shock reflects a status of no harmonization between the representative agent and the regulator. The absence of the shock $z_t = 1$ $(\forall t)$ is interpreted as the rolling-rule regime, whereby the agent and the regulator smooth investment in environmental quality.

The model is calibrated for quarterly data, where $r = 4\%$, $\beta = 0.96$, $\delta_k = 0.06$, $\delta_E = 0.1$, $\alpha = 0.35$, $\rho = 0.8$ and $\phi = 0.1$. We assume that depreciation in the stock of environmental quality is exactly equal to the fraction of degradation due to consumption. We perform sensitivity analysis regarding the δ_E and ϕ parameters in section 4. The model is solved as a discounted stochastic linear regulator problem. See Ljungqvist and Sargent (2000, pp.56-75) for details and stability conditions.

Mapping the original model into the linear quadratic regulator produces,

$$\max E_0 \sum_{t=0}^{\infty} \beta^t \{x_t' R x_t + u_t' Q u_t\} \qquad 0 \leq \beta \leq 1 \tag{19}$$

subject to

$$x_{t+1} = A x_t + B u_t + C \epsilon_{t+1} \tag{20}$$

The Bellman equation for this problem is

$$v(x) = \max_{u} \{x_t' R x_t + u_t' Q u_t + \beta E\left(x' P x\right) + \beta d\} \tag{21}$$

where the scalar d is given by $d \equiv \beta(1-\beta)^{-1} tr P \Sigma$ and Σ is the variance covariance matrix of the random stochastic process. In the original form, the utility is not quadratic. Therefore, we transform it as described in McGrattan (1994) and Anderson, Hansen, McGrattan and Sargent (1996). The transformation is carried by approximating the momentary utility function $r(v_t)$ by $v_t' M v_t$. We approximate $r(v_t) \equiv x_t' R x_t + u_t' Q u_t$ at the steady state point $\overline{v}$ by a Taylor series approximation.[11] The value function for this problem is $v(x) = x' P x + d$, where P is the unique negative semidefinite solution to the discounted algebraic matrix Ricatti equation,

$$P_{j+1} = R + \beta A' P_j A - \beta^2 A' P_j B (Q + \beta B' P_j B)^{-1} B' P_j A \tag{22}$$

For the state vector $x \equiv [1 \; k' \; E' \; \ln z']$, the limit of iterations is P. A feature of this problem is that the feedback solution $F = \beta(Q + \beta B'P'B)^{-1}B'PA$ that characterizes the optimal policy $u_t = -Fx_t$ is identical to the nonstochastic version of this problem. This certainty equivalence principle assures that the conditional mean of the invariant distribution to which the solution tends in the long run equals to the steady state deterministic version of out model (for the proof, see Sargent et al. (2000, pp. 57-58)).

Table 1: **Simulated Correlations**

$c(t)$	$E(t)$	$y(t)$	$m(t)$
1.000	0.337	0.299	-0.392
	1.000	0.965	0.708
		1.000	0.756
			1.000

The system has a solution whenever the eigenvalues of the matrix $A - BF$ are bounded in modulus by $1/\sqrt{\beta}$. For our problem, the eigenvalues are 0.888, 0.979 and 0.783. All are bounded by 1.0206. Given that R is negative definite and that (A, B) are stabilizable, then the system is stable (Sargent et al. (2000, Theorem, p. 61)).

Figures1 and 2 illustrate the impulse responses for environmental quality due to a shock in environmental maintenance. Figure 1 illustrates the impulse responses with different values of ϕ. Figure 2 illustrates the impulse responses with two different values for the depreciation of environmental quality δ_E. From Figure 1, the smaller the ϕ value, the lesser is the effect of the shock. The effect of the shock peaks after 10 quarters and then diminishes faster for smaller values of ϕ. The stronger the degradation of environmental quality, the more persistent is the effect of the shock.

Figure 2 shows that for smaller depreciation values for environmental quality, the effect of the shock is greater in terms of magnitude and persistence.

It is also desirable for the society to exhibit low values for ϕ and δ_E. Since for most environmental quality indices, δ_E reflects a biological cycle or natural phenomenon, therefore, the only parameter that can be reduced by setting up regulations is ϕ. A lower value for ϕ implies a lesser persistence effect on environmental quality from an increase in maintenance spending.

3 Welfare Costs

Does business cycles matter? Lucas (1987, chapter III) derived and computed the potential welfare benefits of business cycle stabilization. That study proposed to measure the cost of business cycles in terms of proportional upward shift in the consumption process that would be required to make a representative consumer indifferent between its random consumption allocation and nonrandom consumption allocation with the same mean. Lucas' calculation suggested only small benefits from reducing the volatility of consumption. Lucas reported that eliminating aggregate consumption variability would be equivalent in terms of utility to an increase in average consumption of \$8.50 per person (one tenth of one percent of 1983 consumption).

Lucas (1987, p. 27) argued that "I want to propose taking these numbers seriously as giving the order-of-magnitude of the potential marginal social product of additional advances in business cycle theory - or more accurately, as a loose upper bound, since there is no reason to think that eliminating *all* [original emphasis] consumption variability is either feasible or desirable objective of policy." Here, the model is setup such that the only source of variability is due to variability in the investment in environmental quality. Therefore, we compute the benefit from eliminating all consumption variability due to regulators discretion in adopting environmental policy regulations. These benefits will give an upper bound for welfare costs to eliminating discretion in environmental quality policies and adopting a stable form of the rolling-rule regime. Evidence of high (low) welfare costs - due to consumption variability - is a clear indication that a stable rule for regulators is favorable (unfavorable) to adopt.[12]

Using preferences that distinguish between risk aversion and intertemporal elasticity of substitution, Obstfeld (1994) found higher costs. Allowing for habit-formation preferences, Campbell and Cochrane (1999) reported larger costs of business cycles. Other examples include: Dolmas (1998); Krusell and Smith (1999); Otrok (2001) and Tallarini (2000). Depending on the parameters used, these studies found gains ranging from one to twenty percent of consumption. Recently, Barlevy (2001, p. 28) reported evidence of substantial welfare effects through fluctuations affecting the growth rate of consumption. The study proposed that business cycles matter in terms of welfare costs because they affect growth by half of one percentage point.

Following Gomes, Greenwood and Rebelo (2001, p. 136), denote the long-run mean and variance of a stationary aggregate consumption $\widetilde{c}$, by $\bar{\widetilde{c}}$ and $\sigma^2_{\widetilde{c}}$, respectively. For consumption, the expected lifetime utility can be approximated by

$$E\sum_{t=0}^{\infty}\beta^t U(\widetilde{c}_t) \simeq \frac{1}{1-\beta}\left\{U(\bar{\widetilde{c}}) + U_{11}(\bar{\widetilde{c}})\,\frac{\sigma^2_{\widetilde{c}}}{2}\right\} \tag{23}$$

The per-period benefit, expressed in units of consumption as a fraction of average consumption, from eliminating the variability in consumption is (Lucas (1987), equation 8),

$$\frac{U_{11}(\bar{\tilde{c}})\,\sigma_{\tilde{c}}^2}{2\,\bar{\tilde{c}}\,U_1(\bar{\tilde{c}})} = \frac{\bar{\tilde{c}}\,U_{11}(\bar{\tilde{c}})}{2\,U_1(\bar{\tilde{c}})}\,\frac{\sigma_{\tilde{c}}^2}{(\bar{\tilde{c}})^2} = \frac{1}{2}\,\gamma\,\left(\frac{\sigma_{\tilde{c}}}{\bar{\tilde{c}}}\right)^2 \tag{24}$$

where γ is the coefficient of relative risk aversion. For the logarithmic utility, the potential welfare benefits from eliminating the variability in aggregate consumption is half of the square of the coefficient of variability. The potential welfare benefits from eliminating the variability are contained in Table 2.

Table 2: **Variability and Benefits - PHI**

	$\phi = 0.0$	$\phi = 0.1$	$\phi = 0.2$
$\sigma_{\tilde{c}}/\bar{\tilde{c}}$	0.921	0.897	0.875
$\frac{1}{2}\,(\sigma_{\tilde{c}}/\bar{\tilde{c}})^2$	0.424	0.402	0.382

These are relatively larger numbers than suggested by Lucas (1987). Given the solution to the linear optimal regulator problem, we assess the actual benefits - including consumption and environmental quality - from eliminating business cycles fluctuations by computing the ratio below[13],

$$\frac{v_s(x)}{v_{ns}(x)} = \frac{x'Px + d}{x'Px} \tag{25}$$

where $v_s(x) = x'Px + d$ and $v_{ns}(x) = x'Px$ denote the Bellman value function with random stochastic shocks and nonstochastic shocks, respectively. d is defined as on page 8. For the calibrated model, this ratio is 19 percent. Eliminating business cycles induces a welfare benefit of 19 percent. This result falls within the range reported in the literature. However, the innovation here is the existence of preference towards environmental quality.

4 Local Sensitivity Analysis

Kim and Pagan (1995, pp. 380-381) proposed two approaches (local and global) to sensitivity analysis in computable general equilibrium models. Our focus is on local sensitivity analysis. We computed the "sensitivity elasticities" for the models' parameters. These elasticities are based on the Taylor series expansion of a function of the calibrated parameters $g(\theta)$ around θ^* featured in the model. Formally,

$$g(\theta) \simeq g(\theta^*) + \left[\frac{\partial g}{\partial \theta}\right]_{\theta=\theta^*} (\theta - \theta^*) \tag{26}$$

In terms of proportionate changes,

$$\frac{g(\theta) - g(\theta^*)}{g(\theta^*)} \simeq \sum_{j=1}^{p} \eta_j \left[\frac{(\theta_j - \theta_j^*)}{\theta_j^*}\right] \tag{27}$$

where,

$$\eta_j \equiv \left\{ \left[\frac{\partial g}{\partial \theta_j}\right] \left[\frac{\theta_j}{g}\right] \right\}_{\theta=\theta^*} \tag{28}$$

η_j is the sensitivity elasticity for the jth coefficient. These elasticities are computed numerically by perturbing the coefficients of interest. Table 3 reports the model elasticities, where g is defined as the ratio of the standard deviations of model output to sample GDP.

Table 3: **Elasticities**

	$\phi = 0.0$, $\phi = 0.1$	$\phi = 0.0$, $\phi = 0.1$
η	-0.004	-0.007
	$\delta_E = 0.0$, $\delta_E = 0.05$	$\delta_E = 0.05$, $\delta_E = 0.1$
η	-0.578	-0.247

From the above results, if one changes ϕ by 1 percent in the range [0.0, 0.1], the model implies a change of 0.004 percent in the ratio of the model output standard deviation relative to business cycle data GDP standard deviation. Also, higher sensitivity elasticities are reported for changes in δ_E. These results suggest that the model is robust to changes in the degradation and the environmental depreciation parameters.

5 Conclusions

We proposed and simulated a stochastic dynamic general equilibrium model that incorporates the evolution of environmental quality. The model is solved using linear quadratic dynamic programming by iterations on the algebraic Ricatti equation. Theoretically, the smaller is the forgone fraction of environmental quality due to consumption, the lesser is the effect of a shock in maintenance on environmental quality and on output. The effects of the maintenance shock on environmental quality reaches a peak after 10 quarters and then diminishes. The presented model produces a similar response to the Kuznet curve for environmental quality. This response is a function of the random shock and the model calibration parameters.

The results are essentially insensitive to the values of the parameters' calibration. Sensitivity elasticities are reported and suggest that the model is relatively robust with respect to the values used for $\phi$and δ_E. Conditioned on the parameters used, we show that the potential welfare benefits from eliminating business cycles - due to the variability in regulators' policies - is 19%.

This model can be easily expanded to accommodate endogenous growth (see Jones, Manuelli and Siu (2000)). Most importantly, the model implies an empirical research agenda for the estimation of the environmental-quality calibrated parameters, such as ϕ, δ_E and the intertemporal elasticity of substitution for environmental quality.

Notes

[1] The label "Not In My Back Yard" originated from the movement against the building of nuclear facilities in the neighborhood. See the source at http://www.ucc.ie/cgi-bin/acronym?NIMBY.

[2] Also labelled as the "collaborative model."

[3] Savitz (1999) described feasibility as "wishful thinking."

[4] For a comprehensive treatment of this issue, see the answer provided by Sabel, Fung and Karkkainen (1999).

[5] We thank Curtis J. Eberwein for suggesting the use of the lognormal distribution.

[6] For an excellent and comprehensive development of the representative agent in macroeconomics modeling, refer to Hartley (1997).

[7] The actual numerical value of utility is irrelevant. A change in the utility level provides a measure of the direction of welfare change.

[8] Fullerton (2001) proposed a single model to analyze the distributional effects of eight different types of environmental policies. Designed to generate efficient allocations, the model suggested that for each type of pollutant, a different policy was less costly and more feasible to enact. We abstract from distributional issues by assuming a representative agent.

[9] The marginal rate of transformation equals the sum of agents' marginal rate of substitution.

[10] The space l_∞consists of all sequences $x = (x_1, x_2, ...)$, $x_n \in R$, that are bounded in the norm $\| x \|_\infty = \sup_i \mid x_i \mid$.

This space is very important for the two welfare theorems. The space l_∞ ensures that assumptions 15.3 and 15.5 (Stokey and Lucas [with Prescott] (1989, p. 455)) hold for the preferences and technologies of interest. For infinite horizon stochastic optimal growth models, any space of the l_p spaces other than l_∞ causes serious difficulties. Stokey and Lucas (1989) defined this space (pp. 447-449), emphasized its role in the two welfare theorems (pp. 458-460), and explained its extension to stochastic growth models (p. 462).

[11] Approximating the utility using the two dimensional version of Taylor's theorem for $U(c, E)$ yields,

$$\begin{aligned} U(c,E) \simeq\ & U(c^*, E^*) + U_c|_{c^*}(c - c^*) + U_E|_{E^*}(E - E^*) \\ & + \frac{1}{2}\left[U_{cc}|_{c^*E^*}(c - c^*)^2 + 2U_{cE}|_{c^*E^*}(c - c^*)(E - E^*) + U_{EE}|_{c^*E^*}(E - E^*)^2\right] \\ & + R_2 \end{aligned}$$

where the derivatives are evaluated at the steady state.

[12] A caveat from such an approach is the absence of a metric by which one can measure 'closeness'. Indeed, a solution for such an absence is to be sought in future studies.

[13] Gomes, Greenwood and Rebelo (2001, p. 137) used a similar ratio to assess the

compensating variation between the two - stochastic and nonstochastic - economies.

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6 Appendix

6.1 Utility Specification

Since the extent of the effect of the shock depends on the intertemporal substitution of environmental quality, this appendix defines and emphasizes the role of elasticities. For the general momentary utility function,

$$U(C,E) = V(C).C(E) = \frac{1}{1-\sigma}C^{1-\sigma}.\frac{1}{1-\sigma_1}E^{1-\sigma_1} \tag{29}$$

we have,

$$\begin{array}{ll} U_C = C^{-\sigma}V(E) > 0 & U_E = E^{-\sigma_1}V(C) > 0 \\ U_{CC} = -\sigma C^{-1-\sigma}V(E) < 0 & U_{EE} = -\sigma_1 E^{-1-\sigma_1}V(C) < 0 \\ U_{CE} = U_{EC} = C^{-\sigma}E^{-\sigma_1} > 0 & \end{array} \tag{30}$$

The elasticities of marginal utility U_C with respect to C and E are,

$$\zeta_{U_cC} = \frac{U_{CC}C}{U_C} = \frac{-\sigma C^{-1-\sigma}V(E)C}{C^{-\sigma}V(E)} = -\sigma \tag{31}$$

$$\zeta_{U_cE} = \frac{U_{CE}E}{U_C} = \frac{C^{-\sigma}E^{-\sigma_1}E}{C^{-\sigma}\frac{1}{1-\sigma_1}E^{1-\sigma_1}} = 1-\sigma_1 \tag{32}$$

The intertemporal elasticity of substitution in consumption equals σ. As σ increases (approaches 1, i.e. logarithmic), the decrease in U_C is more rapid in response to an increase in C, and the consumer is less willing to accept deviations from a uniform pattern of consumption.
The elasticities of marginal utility U_E with respect to C and E are,

$$\zeta_{U_EC} = \frac{U_{EC}C}{U_E} = \frac{C^{-\sigma}E^{-\sigma_1}C}{E^{-\sigma_1}\frac{1}{1-\sigma}C^{1-\sigma}} = 1-\sigma \tag{33}$$

The intertemporal elasticity of substitution in environmental quality equals σ_1, as shown by,

$$\zeta_{U_EE} = \frac{U_{EE}E}{U_E} = \frac{-\sigma_1 E^{-1-\sigma_1}V(C)E}{E^{-\sigma_1}V(C)} = -\sigma_1 \tag{34}$$

6.2 Lognormal Distribution

The lognormal distribution is a special case of Gilbrat's distribution, whenever S=1 and M=0. The pdf $p(x)$ and cdf $d(x)$ are,

$$p(x) = \frac{1}{Sx\sqrt{2\pi}}\exp\left\{\frac{(-\ln x - M)^2}{2S^2}\right\} \tag{35}$$

$$D(x) = \frac{1}{2}\left[1+\operatorname{erf}\left(\frac{\ln x - M}{S\sqrt{2}}\right)\right] \tag{36}$$

where

$$\mathrm{erf}(z) = \frac{2}{\sqrt{\pi}} \sum_{n=0}^{\infty} \frac{(-1)^n z^{2n+1}}{n!(2n+1)} \tag{37}$$

The mean μ and variance σ^2 are given by,

$$\mu = \exp\left(M + \frac{S^2}{2}\right) \tag{38}$$

$$\sigma^2 = \exp\left(S^2 + 2M\right)\left(\exp\left(S^2\right) - 1\right) \tag{39}$$

Examples of variates which have approximate lognormal distribution include: the size of silver particles in a photographic emulsion, the weight and blood pressure of humans and the number of words written in sentences by George Bernard Shaw. For the use of the lognormal in economics, refer to Aitchison, J. and Brown, J. A. C. (1957) *The lognormal Distribution, with Special Relevance to its use in Economics.* New York: Cambridge University Press.

7 Figures

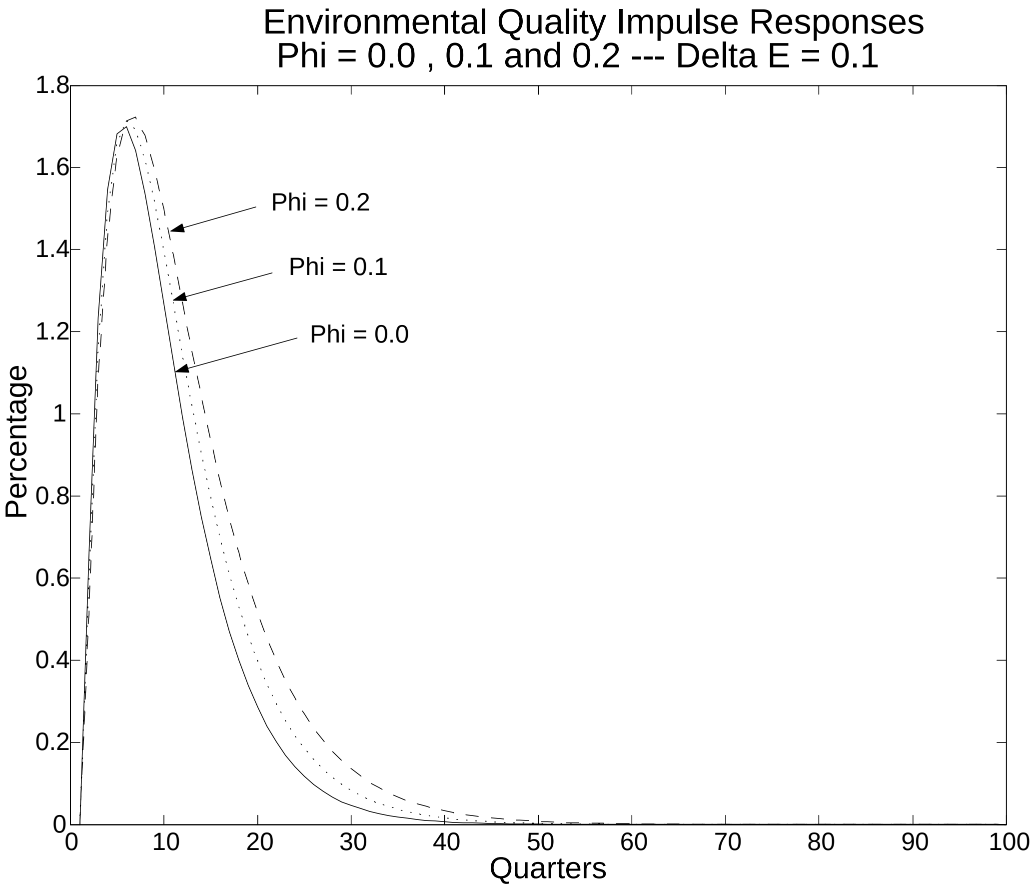

Figure 1: E-Quality Impulse Responses - PHI

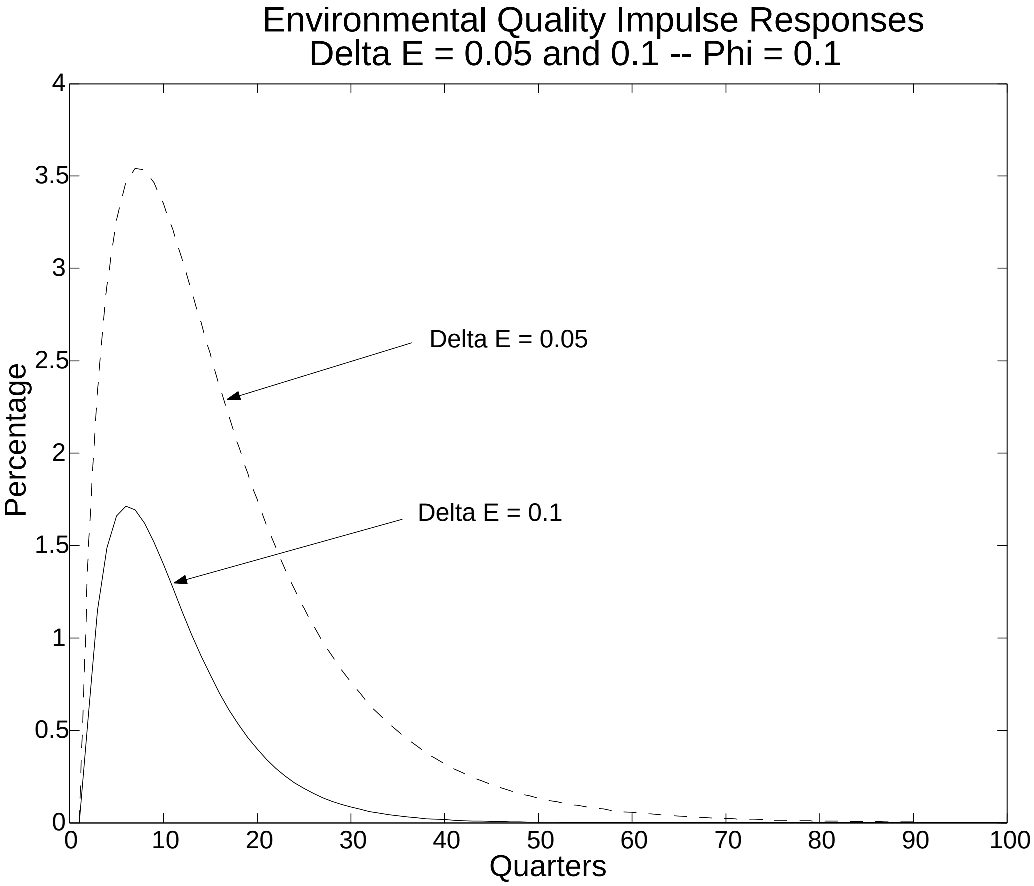

Figure 2: E-Quality Impulse Responses - Delta E